MUSICA UNIVERSALIS:
FROM THE LAMBDOMA OF PYTHAGORAS
TO THE TONALITY DIAMOND OF HARRY PARTCH

by

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An independent research project
submitted in partial fulfillment of the
degree of Master of Arts
Montclair State University, Montclair, NJ

2013

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Independent Study, Theory/Composition
Course Paper
5/1/2013

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ABSTRACT

Seldom does it occur to the modern musician to question the system in which he/she works. For most Western musicians, twelve-tone equal temperament is simply a fact of life, the undisputed foundation of our art. Although microtonal composers and theorists do posit their alternative approaches, rarely is the question asked: what originally compelled us toward a twelve-tone system? The answer is to be found in the intersection between music and philosophy. The number twelve is directly connected to astrology, the signs of the zodiac. Each tone was meant to represent a position of the sun in the heavens, a connection to *musica mundana* or *musica universalis*, the "music of the spheres," an idea we trace back to the philosopher who supposedly discovered the relationship between tone and number: Pythagoras of Samos (ca. 570—ca. 490 BCE). Pythagoras believed that number was the essence of the universe, and that understanding the numerical proportions of harmonics was the key to understanding the universe—of literally unifying our consciousness with the mind of God. To this end he discovered or devised mathematical concepts such as the *mensa pythagorica*, a table of ratios derived from the study of the monochord, also known as the *Lambdoma.* The *Lambdoma* is

considered by modern Pythagorean researchers to be one of the cornerstones of his philosophy.

Harry Partch, pioneer of the modern microtonal movement, studied the art and philosophy of the ancient Greeks at length, tracing what he learned from Helmholtz's *On the Sensations of Tone* and Kathleen Schlesinger's writings on Greek music back to the original sources, and what he perceived to be the original Greek aesthetic. Though it is doubtful that Partch studied the Pythagorean Lambdoma at any length, or ever discovered the writings of neo-Pythagoreans such as Albert von Thimus or Hans Kayser, the configuration and underlying concept of Partch's Tonality Diamond bears so much similarity to the Lambdoma that it must be regarded as a further development and distillation of the original abstract idea into practical application and corporeal form. By examining this connection we trace a line from the modern work of a twentieth century American composer back to a mathematical construct at the heart of ancient Pythagorean (and hence Platonic) teaching and philosophy, and explore the question: is any interpretation or study of the works of Plato—particularly *Republic* and *Timaeus*—complete without an understanding of music, microtonality, and the musical nature of Plato's numerical metaphors? And, conversely: is there any Western music theory without the concept of *musica universalis*?

ACKNOWLEDGEMENTS

I wish to thank:

Chuck Corey, who stepped up to help me through the end run, and who has always generously shared what he knows about just intonation;

Nate Liberty, always there to help, a constant reassuring presence in the Institute;

Joscelyn Godwin, whose books I have enjoyed for years, and who has endured my nagging emails;

Barbara Hero, who was nice enough to welcome me into her home while recuperating from surgery, and who generously shared her time and work, and introduced me to the PLHK;

Werner Schulze, who granted me time via telephone and who has also endured my insistent questions via email;

The rest of my professors at Montclair State University—Ting-Ho, Darren O'Neill, Bob Aldridge—who have made my re-entry into collegiate life such an overwhelming pleasure;

Krista Spradlin, for serving as my German translator;

My good friend Dr. Scott Paul, for turning me on to Jacob Needleman;

My mother Janice, for continuing to be there;

My wife Pana, for unending support and love, who graciously suffers my babbling about all this math she doesn't understand;

And finally my daughter Gaia, just for existing.

This paper was submitted for credit as part of the coursework for a Master of Arts Degree in Music Composition, but it also serves as my first attempt at writing on a subject that has been a matter of personal obsession for years…and, the world be warned, it will probably not be my last attempt.

Last but not least, I wish to dedicate this paper to the memory of Dean Drummond, composer and steward of the Partch Instrumentarium, who passed away on April 13, 2013, while this paper was being prepared. Dean was a mentor to me, and I'll be forever grateful to him for his wisdom and for making it possible for me to know the joy of the Partch instruments. I came to MSU specifically because of the program he was offering there, and my time with him at the school was all too brief. Rest in peace, dear teacher, and thank you for everything.

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APOLOGIA

Music, as high art, is always the expression of a philosophy. Composers are all amateur philosophers, whether or not they have any experience with the true path of philosophy, Western or otherwise. We can rest assured that all our greatest composers had something to say about the world that was deeply personal, motivated by a combination of political, religious, and intuited beliefs, all of which were the products of cultural conditioning, education, and experience. Even if the subject matter or narrative of a musical piece was not original, and mostly chosen for its evocative emotions or historic significance, what underlies the textual choices are the tools of expression: harmonies, melodies, instrumental colors, all of which will be arranged by the creative mind in order to devise a framework of reality unique to the consciousness of the creator.

In spite of this, music and philosophy exist today as separate disciplines. The average Western musician, though he/she may seethe with passion for his/her craft, does not often question the meaning of the tools of expression he/she uses every day. What he/she chooses to create from the means at his/her disposal may be the product of personal philosophy; still, what about the choices made by theoreticians of antiquity, the decisions that isolated one tonal identity from another, and that led eventually to a system of twelve divisions of an octave, tuned so as to make every semitone equidistant? What about the decision to move away from modalism to a system based primarily in major-minor tonality? But to ponder such questions is to move away from the mere academic study of such things, to set aside the conservative approach of merely maintaining a status quo, and to inquire into the heart of the art, the underlying reason for what we do. Why does music matter? Why does philosophy matter? How are they connected?

> To promote a youthful vitality in music we must have students who will question every idea and related physical object they encounter. They must question the corpus of knowledge, traditions, and usages that give us a piano, for example—the very fact of a piano; they must question the tones of its keys, question the music on its rack, and, above all, they must question, constantly and eternally, what might be called the philosophies behind device, the philosophies that are really responsible for these things.[1]

> It has become more and more clear that philosophy, real philosophy, is an imperative need in our lives and in the life of our world. The word *philosophy* was coined in ancient Greece, and throughout the ancient world it was understood as a way of living and not merely as a mental, academic exercise. It was a way of conducting one's life in which the contemplation of great ideas and the questions they evoke again and again reminded one that we men and women have within ourselves a divine quality of consciousness and love, and that human life on earth is in chaos because this truth about ourselves has been forgotten and because access to this reality in ourselves is blocked…Real philosophy was born as the effort of the truly independent mind—independent of physical and psychological desires and cultural conditioning—to open to the great Independent Intelligence of the Universe and direct us toward the path of awakening self-knowledge.[2]

This word *philosophy* was actually brought to the world by the mysterious man who stands at the beginning of this inquiry: Pythagoras. In his time, learned men were referred to as *sages*, a term that literally means "one who knows." More modest in his terminology, Pythagoras defined himself as "one who is trying to find out."[3] The distinction between these descriptions is vital, because great ideas and the discoveries they lead to have a way of hardening into dogma, forming systems that discourage further adventuring or deviations from accepted norms. Yet Pythagoras declared the mission of the philosopher: to remain open, to keep asking the important questions, to not be satisfied, to remain a student.

Alfred Whitehead wrote that all Western philosophy is merely a collection of "footnotes to Plato."[4] If Plato is the father of all our philosophy, then the grandfather is Pythagoras, and both men are important to our discussion. Teachings attributed to Pythagoras echo all throughout Plato's writings. "Attributed" is the only appropriate

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[1] Harry Partch, *Genesis of a Music* [New York: Da Capo Press, 1974], xv.
[2] Jacob Needleman, *The Heart of Philosophy* [New York: Tarcher/Penguin, 1982], x.
[3] Manly P. Hall, *The Secret Teachings of All Ages* [New York: Tarcher/Penguin, 2003], 192.
[4] Kenneth Sylvan Guthrie, *The Pythagorean Sourcebook and Library* [Grand Rapids, MI: Phanes Press, 1988], 19.

word because Pythagoras left no documentation behind, and all that remains of him are fragments written hundreds of years after his death, and not by any who personally knew him. His life has become the stuff of legend, filled with myths of divine power and Christ-like presence. In his day he was famous not for mathematics and science but for teachings about reincarnation, asceticism, and ritual; he was also rumored to have a thigh made of gold and the ability to bi-locate himself. The teachings about mathematics that are ascribed to him remain the subject of some controversy, for although Plato and Aristotle both spoke of "Pythagorean" ideas they did not cite him directly in doing so, and mention him only in passing as "the founder of a way of life"[5]—a reference to Pythagoras' teachings on fasting, vegetarianism, the transmigration of souls, etc.

Since the exact origin of so-called "Pythagorean" ideas is no longer verifiable (if it ever truly was), the less controversial choice in the use of the term is to let it stand for a certain way of viewing the universe, where the tools of science do not merely stand as the means of problem solving, but can also have larger meaning to the mind. Number—in Pythagorean thinking, the essence of all things—becomes not just *quantity* but also *quality*; One is not just a single unit, but an archetype, an idea, The One, Unity, Primum Mobile, First Thought, First Impulse, "Let there be light!" etc. And even if intellectuals must insist upon all metaphysical associations with number being tossed over the side, the fact remains that all numbers can be expressed as *tone*, and therefore that *quality* of number is a physical fact that does not go away even if the scholar chooses to separate science from theology.

[5] Stanford Encyclopedia of Philosophy. "Pythagoras." http://plato.stanford.edu/entries/pythagoras/ [accessed March 7, 2013].

INTRODUCING…THE LAMBDOMA

The focus of our investigation is a configuration of numbers in grid formation that goes by two names: the Pythagorean Table, otherwise known as the *Lambdoma.* Far more than a simple mathematical table, the *Lambdoma* is a grand philosophical idea, the expression of a universal truth. In the words of Joscelyn Godwin:

> …the Pythagorean Table, whatever its origin, is an incomparable aid to speculative music, as a means toward symbolic explanation and possible illumination concerning cosmic and metaphysical realities. The Table is an image of the Universe. If extended to mathematical infinity it would contain every rational fraction and integer. Each one of these, expressed by a numerator and a denominator, is the product of an intersection between an overtone and an undertone row, i.e., every tone occurs as a member or one row of each type. If each is taken to represent one of the beings in the Universe, this dual origin emphasizes each being's dependence on a primordial duality, the initial split of the Lambdoma. One might say that whenever the two forces of contraction and expansion meet and are held in some proportional balance, a being arises—and a tone is sounded. Every being is both number and tone; both quantity and quality; both existence and value. All have the same root: the originating 1/1 tone that represents God the Creator.[6]

The Pythagorean Table
(12 x 12 grid)

1/1	2/1	3/1	4/1	5/1	6/1	7/1	8/1	9/1	10/1	11/1	12/1
1/2	2/2	3/2	4/2	5/2	6/2	7/2	8/2	9/2	10/2	11/2	12/2
1/3	2/3	3/3	4/3	5/3	6/3	7/3	8/3	9/3	10/3	11/3	12/3
1/4	2/4	3/4	4/4	5/4	6/4	7/4	8/4	9/4	10/4	11/4	12/4
1/5	2/5	3/5	4/5	5/5	6/5	7/5	8/5	9/5	10/5	11/5	12/5
1/6	2/6	3/6	4/6	5/6	6/6	7/6	8/6	9/6	10/6	11/6	12/6
1/7	2/7	3/7	4/7	5/7	6/7	7/7	8/7	9/7	10/7	11/7	12/7
1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8	9/8	10/8	11/8	12/8
1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	9/9	10/9	11/9	12/9
1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	10/10	11/10	12/10
1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11	11/11	12/11
1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	12/12

Our mission here is twofold: first, we must understand the philosophical implications of this table, as well as the fact that, according to Pythagorean teaching, this table is the source of all mathematical understanding of tonal relationships; and second, we will trace the study and evolution of the table into a configuration known as the

[6] Joscelyn Godwin, *Harmonies of Heaven and Earth* [Rochester, VT: Inner Traditions, 1995], 179.

Tonality Diamond (shown below), invented in the twentieth century by American composer Harry Partch. All these two grids may appear to have in common are the fact that they are composed of ratios; but with knowledge of both tables as musical realities, the second will be revealed as a distillation of the first.

Tracing the line of descent from one matrix to the other reveals a unity of science, philosophy, and art that has fallen into obscurity; yet it remains a living heritage for those passionate enough to trace music theory to its true roots. An attempt at revival began in the nineteenth century and re-awoke in the twentieth through the efforts of musico-philosophers on two continents—who have not all been entirely aware of each other. Following our introduction to these diagrams we shall examine something of the lives of the men and women at each end of this line of descent. (For a visual of the complete timeline, see Appendix IV.)

Harry Partch's Tonality Diamond

					11/8							
				9/8		11/10						
			7/4		9/5		11/6					
		3/2		7/5		3/2		11/7				
	5/4		6/5		7/6		9/7		11/9			
1/1		1/1		1/1		1/1		1/1		1/1		
	8/5		5/3		12/7		14/9		18/11			
		4/3		10/7		4/3		14/11				
			8/7		10/9		12/11					
				16/9		20/11						
					16/11							

CHAPTER 1: PYTHAGORAS

HIS LIFE

There are no written accounts of Pythagoras' life (ca. 570-ca. 490 BCE) before the fragments dating about 150 years after his death. The first complete account of his life comes from Iamblichus (245-325 CE), a Syrian Neoplatonist philosopher who venerated Pythagoras as a divine being, and thus the facts are well mixed with the mythical; at times Iamblichus seems to be attempting to present Pythagoras as a rival for Jesus. Manly P. Hall (*The Secret Teachings of All Ages*) claims that during his early travels Pythagoras was initiated into the mystery schools of the Egyptians, Babylonians, and Chaldeans[7], and the earliest surviving writings about his life appear to bear this out (Herodotus, Isocrates). Other than that, little can be verified other than that he was born on the Greek island of Samos, immigrated to the Italian city of Crotona at about age 40 when the tyranny of the ruler Polycrates became intolerable and later escaped to Metapontum when popular opinion turned against him and his disciples (ca. 510 BCE). Accounts of his death do not agree. Some say he was assassinated; others claim him to have passed peacefully, if not happily, in Metapontum.[8]

Many believe that even the mathematical principles bearing the stamp of Pythagoras were actually invented later by successive generations of disciples, and so the surviving mystical teachings are thought to have more historical validity. Two concepts are crucial to this discussion: first, the dictum "All is Number;" and second, "the music of the spheres," also known as *musica universalis* or *musica mundana*, the idea that the

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[7] Hall, 192.
[8] Stanford Encyclopedia of Philosophy. "Pythagoras." http://plato.stanford.edu/entries/pythagoras/ [accessed March 7, 2013].

motions of the planets create tones that harmonize with one another and that this heavenly harmony is the model for earthly harmony, a model which applies not only to art but to individual and societal life. What is essential to an understanding of this second concept is the relationship of number to tone. We will start, however, with the Pythagorean assertion regarding the true nature of numbers.

NUMBER AS ESSENCE

> Where Pythagoras developed his interest in number we do not know, although it is likely that he was not the first to be concerned with its sacred or metaphysical dimension. What we do know is that a metaphysical philosophy of Number lay at the heart of his thought and teaching, permeating…even the domains of psychology, ethics and political philosophy. The Pythagorean understanding of Number is quite different from the predominately quantitative understanding of today. For the Pythagoreans, Number is a living, qualitative reality which must be approached in an experiential manner. Whereas the typical modern usage of number is as a sign, to denote a specific quantity or amount, the Pythagorean usage is not, in a sense, even a usage at all: Number is not something to be *used;* rather, its nature is to be *discovered.*[9]

Even the most scientifically minded of us cannot completely shrug off the idea of number being at the heart of our reality. Consider the Periodic Table of Elements. What distinguishes one atomic substance from another, if not number? For Pythagoras and his brethren, number was First Principle, superseding all else; and number was, and remains, *tone.* This is not just in the sense that tone is *frequency*, as in vibrations per second measurable in Hertz (Hz). Tone-number correspondence is made apparent by the study of the *harmonic series*, a natural aural phenomenon that defines all intervallic tonal relationships as numerical.

If we experience number as tone, there is no denying that number has a quality of character, because that is definitely how we experience tone. Tone is capable of eliciting emotional responses, physical sensations, imagery, ecstatic states of consciousness, and

[9] Guthrie, 20.

more. Shamans employ tone to facilitate journeys into non-ordinary reality. Some people, such as the great French composer Olivier Messiaen (1908-1992), report experiencing tones as colors. Others experience them as smells, tastes, temperatures, feelings, etc. We have preferences for different kinds of musical sounds, according to what resonates within us on both personal and universal levels.

In considering the worldview of number as *quality* it is important to remember that in the time of the Pythagoreans the mathematicians and the philosophers were the same men; arithmetic and numerology were one discipline, not two. Gradually through the centuries science has demanded freedom from theology to escape the tyranny of metaphysical concepts that have hardened into dogma, but for the ancients every number was a universal principle and therefore connected to ideas, myths, and gods. For more about Pythagorean belief regarding the nature of Number, see Appendix I.

MATH AS MUSIC

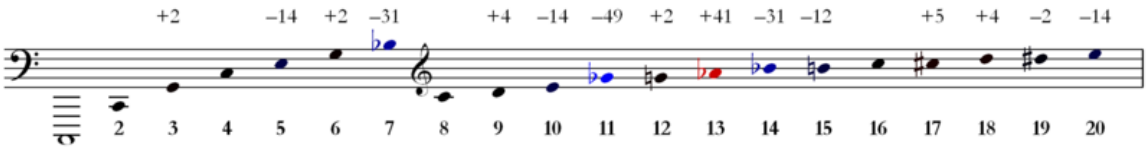
One of the discoveries Pythagoras is traditionally credited with is the relationship between number and tone. In his *Manual of Harmonics*, the Pythagorean philosopher Nicomachus of Gerasa (ca. 60-ca. 120 CE) tells the first written account of the legend of the Harmonious Blacksmith, a story still told today about how Pythagoras accomplished this signature achievement.

> One day he was deep in thought and seriously considering whether it could be possible to devise some kind of instrumental aid for the ears which would be firm and unerring…while thus engaged, he walked by a smithy and, by divine chance, heard the hammers beating out iron on the anvil and giving off in combination sounds which were most harmonious with each other, except for one combination. He recognized in these sounds the consonance of the octave, the fifth and the fourth. But he perceived that the interval between the fourth and the fifth was dissonant in itself but was otherwise complementary to the greater of these two consonances. Elated, therefore, since it was as if his purpose was being divinely accomplished, he ran into the smithy and found by various experiments that the difference of sound arose from the weight of the hammers, but not

> from the force of the blows, nor from the shapes of the hammers, nor from the alteration of the iron being forged.[10]

Nicomachus goes on to describe in detail how Pythagoras went home and experimented with weights and various proportional lengths of string until he fully identified the mathematical relationships involved. This story was passed on practically unaltered by various other writers (Adrastus, Gaudentius, Censorinus, Iamblichus, Macrobius, Boethius, and others), in spite of incongruities in the later descriptions. "One has only to try duplicating the experiment detailed by Nicomachus in this chapter to discover that it does not work, that the results adduced by Nicomachus simply cannot be obtained,"[11] translator Flora Levin comments in her 1994 edition of the work. The ongoing endurance of the tale attests to its simple effectiveness in explaining this monumental realization which formed the foundation of all Western music theory as we know it. Prior to this discovery tonality was purely a matter of intuition, done by the ear; Pythagoras showed us that mathematical relationships and tonal relationships were one and the same.

Fig 1: The Harmonic Series



The pitches of the harmonic series, based on the primary pitch of C three octaves below middle C. Numbers above the staff show differentiation in *cents* (a method of tonal measurement: one *cent* is a 100th of an equal-tempered semitone) from pitches tuned in the Western system of 12-tone equal temperament (12-TET). Note that certain partials (7, 11, 13, 14) differ so greatly from 12-TET pitches as to be impossible to even approximate in standard tuning. (Image reproduced from Wikipedia.org)

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[10] Flora R. Levin, *The Manual of Harmonics of Nicomachus the Pythagorean* [Grand Rapids, MI: Phanes Press, 1994], 83.
[11] Ibid., 87.

CHAPTER 2: HARRY PARTCH

Harry Partch (1901-1974) also qualifies as a somewhat enigmatic figure by virtue of his complete lack of academic credentials, and an artistic contribution to Western culture that, while historic, remains far outside the mainstream. Partch remained committed to a unique vision through the course of his life in spite of extreme hardships, some of which were the result of what could be regarded as highly impractical choices. Being largely self-taught, Partch made attempts early on at a career in performance, but was too full of antagonizing questions to fit comfortably into the mainstream. Early exposure to the music of China and Native America may have influenced him insofar as how he heard music, but he also spent long periods of time in isolation as a young man, and much of mainstream music never seemed "in tune" to him.

Oddly enough, Partch was correct; modern music is actually not "in tune." Modern music is governed by the rules of piano tuning, which requires a "tempering" of tuning in order to sound uniform. Using a system that equalizes the distances between all semitones may have its advantages, but it is not "in tune;" all intervals on a piano, with the exception of the octaves, are slightly detuned from the harmonic series. Perfect 5ths on a piano are slightly flat, major 3rds are slightly sharp. It is not possible to tune a piano to the harmonic series and have all intervals sound the same across all the keys. While Partch intuited this to some degree, he did not envision a viable alternative until discovering a certain book at the Sacramento Library in 1923, an English translation of German scientist Hermann von Helmholtz's *On the Sensations of Tone.*

HELMHOLTZ

Hermann Ludwig Ferdinand von Helmholtz (1821-1894), a famous scientist and acoustician, studied medicine at Friedrich Wilhelm Medical Institute in Berlin (M.D., 1843) and learned piano. He served as Professor Extraordinary at the Berlin Academy of Fine Arts, Director of Königsberg's Psychological Institute (1849-55), Professor of Anatomy and Physiology at University of Bonn (1855-58) and University of Heidelberg (1858-71). He was also a Professor of Physics at University of Berlin in 1871, the first director of Physico-Technical Institute in Berlin (1888), and was ennobled in 1882. He published *Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* in 1863, translated by A. Ellis as *On the Sensations of Tone as a Physiological Basis for the Theory of Music* in 1875 in London, and published in a New York edition in 1948.

Helmholtz's stated objective was to reestablish the mathematical basis for musical harmony, and in doing so reunify the ancient disciplines. "The horizons of physics, philosophy, and art have of late been too widely separated," he states. "And, as a consequence, the language, the methods, and the aims of any one of these studies present a certain amount of difficulty for the student of any other of them; and possibly this is the principal cause why the problem here undertaken has not been long ago more thoroughly considered and advanced towards its solution."[12]

Partch's discussions of ancient Greek music theorists in *Genesis* do include Plato and Pythagoras, but there is no mention whatsoever of the Lambdoma—unsurprising, since the only existing written authority in his time, Albert Freiherr von Thimus'

[12] Hermann von Helmholtz, *On the Sensations of Tone as a Psychological Basis for the Theory of Music* [London: Longmans, Green, and Co., 1930], 1

Harmonikale Symbolik des Alterthums, had never been translated into English (and it remains so to this day, other than one passage translated by Joscelyn Godwin for his compendium *The Harmony of the Spheres*). One passage in *Genesis*, however, reveals some knowledge of sources other than Helmholtz.

> We might, perhaps, congratulate ourselves that we are not as likely to develop music-minded Platos and Aristotles to condemn us as 'dangerous to the state' because of our musical acts and views, that we are not as likely to exile modern Timotheuses for introducing a few new scale tones (or are we?), and that we do not use or misuse 'semitones' by Chinese-fashioned edicts. But this is a superficial view. On second thought we must wonder why men of the Plato-Aristotle-Ptolemy-Zarlino-Mersenne-General Thompson breadth of interest take no part in musical discussions (aside from the 'appreciation' and 'I-love-music' attitudes), why there is so much unanimity on the content of music at the same time that battles rage over its commas and exclamation marks—that is to say, its 'interpretation'; why there are so few 'searchers' in music as compared with the social and natural sciences, and even with literature…The pressing need is for a realignment of music with rationality—with something for a realistic, philosophic, or scientific bent to operate upon…[13]

This shows that Partch was at least familiar with the theoretical treatises of Gioseffo Zarlino (1517-1590) and Marin Mersenne (1588-1648), two very influential writers whose researches into tuning lay further pavement along the way to the adoption of equal temperament. Zarlino was a Venetian composer, member of the Franciscan Order, author of *Institutioni Harmoniche* (1558) and "the most celebrated music theorist of the mid-16th century."[14] Mersenne, a French theologian, mathematician and fellow Franciscan, is known for his work *Harmonie Universelle* (1637). While both men (as would be natural for men reared under Catholic influence) eschewed all metaphysical correspondences touted by their more polytheistic predecessors, they do not shy away entirely from *musica mundana.*

> Returning now to the soul's music, we will say that it is of two kinds: heavenly and human. The heavenly sort is not only that harmony which is known to exist among things seen and known in

[13] Harry Partch. *Genesis of a Music: an Account of a Creative Work, its Roots and its Fulfillments* [New York: Da Capo Press, 1974], 56.
[14] Encyclopedia Britannica. "Gioseffo Zarlino." http://www.britannica.com/EBchecked/topic/655982/Gioseffo-Zarlino (accessed Apr. 21, 2013)

> the heavens, but is also included in the linkings of the elements and in the changing of the seasons. It is, I say, seen and known in the heavens from the revolutions, distances, and placements of the heavenly spheres, as well as from the aspects, nature, and position of the seven planets: the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn. For it is the opinion of many ancient philosophers, notably of Pythagoras, that the revolution of so vast a machine at such speed could not possibly take place without giving forth some sound. [15]

> One can prove further that the unison is more excellent than the other consonances through Astrology, which sees the consonances in the aspects of the planets. The conjunction is the most powerful and excellent of all aspects, but many deny that it merits the name of aspect, just as they deny that the unison is one of the consonances. In fact, if the conjunction of planets represents the unison—as they hold that the opposition represents the octaves, the trine the fifth, the square the fourth, and the sextile the thirds and sixths—and if the conjunction is more powerful than the other aspects, one may well say that it has a close correspondence with the unison. [16]

There is an interesting anecdote, in Bob Gilmore's biography of Partch to attest to Partch's study of these works, regarding a meeting in England with musicologist and instrument builder, Arnold Dolmetsch. "In Partch's version of the meeting, Dolmetsch, talking effusively about ancient writings on music, refers to Marin Mersenne's *Harmonie universelle* and is astonished when Partch asks him which edition of the book he is referring to, stammering out: 'For *twenty years* I have been talking about Mersenne, and nobody even knows who I am talking about. And now you—you ask me which edition!'"[17] So while Partch appears to be primarily attuned to the practical aspects of these works, he displays a level of scholarly study that indicates exposure to esoteric Pythagorean ideas. As for the somewhat puzzling linkage to General Thompson (inventor of the tommy gun, a soldier who pushed hard for the military to adopt and develop automatic weapons), Partch is presumably stating a connection based upon these men's shared innovative spirit and maverick willingness to buck the trends of their day. Such eccentric correlations serve to make *Genesis* quite an entertaining read.

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[15] Gioseffo Zarlino, "Institutioni Harmoniche." In Godwin, *Harmony of the Spheres*, 206.
[16] Marin Mersenne, "Harmonie Universelle." In Godwin, *Harmony of the Spheres*, 261.
[17] Gilmore, 107.

CHAPTER 3: JUST INTONATION

> What is just intonation, and why would a composer risk alienation from mainstream musical culture in order to use it? The necessity for posing this question upholds one of the aesthetic premises of Harry Partch's work. Partch believed that the uncompromising status of equal temperament in Western culture was an unjust obstruction to creativity, and he sought to raise awareness of this through his writings and his music. It is indeed ironic that the simple mathematical elegance of just intonation confounds so many, while the comparatively complex calculations of equal temperament are accepted with ease. According to Partch, this paradoxical situation resulted from the standardization of pedagogical training, the mass production of musical instruments, the economic interests of the recording and performance industries, and—perhaps most importantly—the stylistic developments in Western music itself. In each case, equal temperament's predominance has ostensibly been passed from one generation to the next without question or comment. The realization of this led Partch to suspect an ideological force behind equal temperament's thrust.[18]

We begin this discussion by acknowledging that the topic of Just Intonation as a tuning system is a rather large and difficult topic, particularly for those who have had no experience with identifying tone in mathematical terms. We shall attempt to explain it in the most concise way possible, along with suggesting Partch's *Genesis of a Music* for further reading. Just Intonation is a term that describes any system of musical tuning that bases its intervallic relationships on the whole-number ratios of the harmonic series, as opposed to tempered systems like our 12-TET. These ratios are used to define pitch-identities. We have no A, B, C-sharp or D-flat in Just Intonation; we begin with one frequency identified as 1/1, and every other pitch is identified by its relationship to the 1/1.

Fig. 2: Just Intervals and their 12-TET Equivalents

Just Interval	12-TET Equivalent	Deviation in Cents
2/1	Octave	0
3/2	Fifth	+2
4/3	Fourth	-2
5/4	Major Third	-14
6/5	Minor Third	+16
9/8	Whole Tone	+4
16/15	Semitone	+12

[18] Brian Timothy Harlan. "One Voice: a Reconciliation of Harry Partch's Disparate Theories" [PhD diss., University of Southern California, 2007], 1.

The birthplace of all human-determined musical law is the nature-determined harmonic series, which is demonstrable either through comparative study of string-lengths or through the phenomenon of *sympathetic resonance*. One does not need varying lengths of string to prove that whole number ratios produce harmonic intervals; all you really need is one string, or your own voice, to experience this law at work. Various methods of singing harmonics abound in the world today, ever since the vocal techniques of Mongols, Tibetan lamas, and Tuvan shamans have entered the global musical sphere. Western forms of the same have been used in the compositions of David Hykes, Toby Twining, Glen Velez, and others. Musicians and scientists alike know of this phenomenon, where the multiples of a frequency sound simultaneous with the baseline frequency and even set other resonant bodies to motion, without any physical contact whatsoever. This phenomenon of sympathetic resonance reveals that our most consonant intervals all originate within the first six harmonics—also referred to as *overtones* or *upper partials*.

Other equally consonant intervals are derived from *undertones* or *lower partials*. But it must be understood that these undertones, while remaining an important tool for establishing tone-identities, are controversial among theorists because of their primarily theoretical nature. No one yet has been able to demonstrate a physical basis for an undertone series; they do not occur naturally. Still, they remain a necessary component of our tuning system, for the sake of proper balance and modal richness. Low overtones produce only "major" intervals, while undertones produce "minor" intervals. In fact, the ratio 4/3, although it is one of the three most important in our system it can only be

expressed as an undertone (the reciprocal of 3/2) or a derivative (the relationship between the 3rd and 4th overtones); there is no way to produce it as an overtone of 1/1.

So let us first identify three different types of ratio/tone-identities. First, there are what Harry Partch referred to as *odentities*—ratios expressing tone-identities that are overtones of 1/1. These ratios, if expressed as fractions, must have a denominator that is a power of 2. If we examine Pythagoras' diatonic scale, we see this holds true of each tone except the 4/3 (1/1—9/8—81/64—4/3—3/2—27/16—243/128—2/1).

Second are the ratios Partch called *udentities*, which are undertones of 1/1. Udentities are easily identified by having powers of 2 as numerators (4/3, 8/7, 8/5, 16/9, etc.). The third are referred to in this work as *derivative* intervals, because they only exist as tone-relationships derived from the harmonic series but not expressible as overtone or undertones. A good example of this is the just minor third 6/5. One way of thinking about this interval is that it is the relationship of the 6th overtone to the 5th overtone, with that relationship then applied to the 1/1.

I should add, however, that Partch did not use these invented terms *odentity* and *udentity* only to refer to tones based off the nexus of 1/1. In Partch's system it is perfectly acceptable to say, for instance, "the 9 Odentity of an 8/5 Unity is 9/5." This is a way of expressing tonal relationships within the framework of a Tonality, which in Partch's terms was a hexachord using any tone as a base representing 1 and involving the upper partials of 3, 5, 7, 9, and 11. But this is beyond the scope of this investigation; we are sticking to the 1/1 as our nexus.[19]

[19] It should be noted that when 4/3 is used as a nexus all the numbers of Pythagoras' diatonic can be found as overtones of 4/3, if we are willing to sacrifice the simplicity of small numbers. Dean Drummond, composer/curator of the Harry Partch Instrumentarium, frequently pointed out that all udentities are expressed as overtones of pitches other than 1/1 if we are willing to deal with more complex numbers, so

Common Odentities: 3/2, 5/4, 7/4, 9/8, 11/8, 15/8, 27/16.

Common Udentities: 4/3, 8/5, 8/7, 16/9, 16/11, 16/15, 32/27.

Common Derivatives: 5/3, 6/5, 10/9, 7/6, 11/6, 9/5, 10/7.

Next, let us distinguish the different methodologies of using numbers to represent tones. These consist of: (1) ratios of string length, (2) ratios of actual frequency, and (3) ratios for the purposes of Just Intonation.

For ratios of string length, we must keep in mind that the shorter the string, the higher the tone. As frequency increases, length decreases. Divisions of 1 produce overtones and multiples of 1 produce undertones, while for ratios of actual frequency the inverse is true. See comparative chart below, for the first five overtones and undertones of G. Henceforth we will use G as our 1/1 rather than C because that is what Harry Partch used, so we will not confuse the issue by doing otherwise.

Fig. 3: String Length/Frequency Comparison Chart

12-TET TONE NAME	STRING RATIO	ACTUAL FREQUENCY RATIO
G	1/1	1/1
G(1)	1/2	2/1
G(-1)	2/1	1/2
D(1)	1/3	3/1
C(-2)	3/1	1/3
G(2)	1/4	4/1
G(-2)	4/1	1/4
↓B(2)	1/5	5/1
↑E𝄫(-3)	5/1	1/5

The reciprocal relationship of frequency to string length is interesting to note, because it gives us the chance to ponder this relationship for greater meaning. Hans Kayser, whose work we will examine in greater depth later, likened this relationship to

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the method of "udentifying" pitches is primarily for simplicity's sake, and to perhaps give access to interesting philosophical speculation (odentity:undentity::yang:yin, etc.).

Einstein's theory of space and time as one continuum rather than discrete concepts (string length representing objects in space, frequency representing time).[20]

Before moving on to discuss the third method, it is necessary to clarify two aspects of translating just intervals into 12-TET pitch classification. Firstly there is the number in parentheses following each pitch, which represents the octave. The G without a number following it is G (just below middle C) on the piano, and G(1) would be the first octave above, while the G(-1) is the first below. Also notice in the 12-TET tone names that some pitches are preceded by the symbols (↑) and (↓). These come from Dean Drummond's method of translating just intervals into 12-TET notation. The method utilizes the standard microtonal division of 100 cents per equal-tempered semitone. Ranges are identified as plus or minus 12.5 cents, giving each possible pitch name a possible range of 25 cents to fall within.

In other words, the pitch ↑G would fall somewhere between 12.5 and 37.5 cents sharper than an equal-tempered G. You can see in Figure 3 that the B of the 5th overtone of G is written ↓B, because the overtone B is 14 cents lower than an equal-tempered B. Therefore it falls into the -12.5 to -37.5 range and is designated with the downward arrow.

Also note the symbol (𝄲) is used to represent a half-sharp, or sharpness by half a semitone (50 cents), while the symbol (𝄳) represents a half-flat. The chart below (Figure 4) demonstrates this.

[20] Kayser, *Textbook of Harmonics* [Sacred Science Institute, 2006], 14.

Fig. 4: Accidentals/Cents Comparison Chart

CENTS	12-TET PITCH
100	G#
87.5	----------
75	↓G#
62.5	----------
50	G(♯)
37.5	----------
25	↑G
12.5	----------
0	G
-12.5	----------
-25	↓G
-37.5	----------
-50	G(♭)
-62.5	----------
-75	↑F#
-87.5	----------
-100	F#

There is only one flaw in this system insofar as the pitches of Partch's Tonality Diamond are concerned. Two pairs of pitches on the Diamond (9/5 & 20/11 and 11/10 & 10/9) end up having the same 12-TET-referenced notation. 9/5 is 18 cents sharper than F, while 20/11 is 35 cents sharper; therefore both fall into the $12.5<x<37.5$ range of ↑F. Similarly, 11/10 is 35 cents flatter than A, while 10/9 is 18 cents flatter, so both fall in range of ↓A. To avoid confusion we will differentiate 9/5 as +↑F and 11/10 as -↓A.

The third method of pitch classification by ratio is that which is employed in the composition of music in just intonation. In this method, pitch names are expressed as fractions that translate into numbers between 1 and 2, such as 3/2 (1.5), 5/4 (1.25), 7/4

(1.75), and so on. Just as in standard composition pitch names are not differentiated by octave, so we do not identify a pitch as a 14/4 when we already know that pitch as a 7/4 (see the Law of Octaves, below). Octave differentiation occurs naturally in notation. We do not even use 2/1, because we already know it as 1/1. So any pitch other than 1/1 has to fit into the equation $1<x/y<2$. Therefore the above graph, once octave duplications are removed, would look like this for the purposes of just intonation:

G	1:1
D	3:2
C	4:3
↓B	5:4
↑E♭	8:5

THE POWERS OF 2 AND 3

It is important to understand the role of the number 2, or the Law of Octaves. We see that our starting frequency of the note G repeats at all powers of 2: 2, 4, 8, 16, and would continue at 32, 64, 128, 256, etc. If you examine this according to frequency in Hertz you see that a doubling of vibrations produces the octave. Any note anywhere in the scale is double the vibration of the octave below, and half the vibration of the one above. This can be applied to all notes anywhere in the series. If a new note-identity occurs, for instance at the 3rd partial where D appears, then 3 multiplied by any power of 2 will also produce a D (3, 6, 12, 24, 48, etc.). Understanding this law is essential for knowing how to octave-reduce the ratios so that they fall within the 1/1-2/1 octave space.

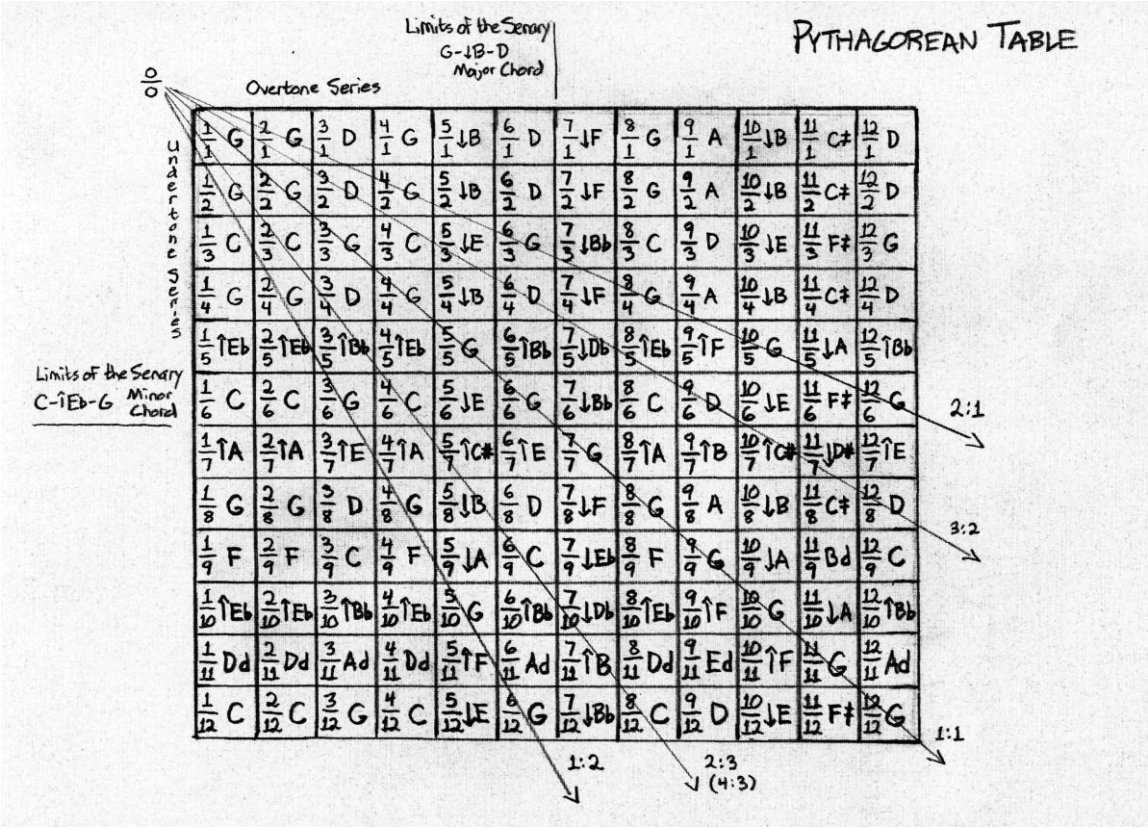
The second rule of importance is the Law of Three. The third overtone produces the note-identity of D, and the interval of what pianists would refer to as a fifth. If we examine all powers of 3 within the series, we see the Circle of Fifths, as it were. 9 is A,

27 is E, 81 is B, 243 is F#, and so forth. Pythagoras determined a diatonic scale through use of the number 3. But this scale has some glaring flaws—the first being the use of the 81st partial for the interval of a major third. This overly large interval is often referred to as a *ditone* (since it is the sum of two 9/8 whole tones) and it is even sharper than an equal tempered major third, which is already sharper than the just 5/4 by 14 cents.

The second flaw in the Pythagorean diatonic is that cycling through twelve 3/2's to get back to an octave of 1/1 does not work, in truth. Starting at a C and going up twelve 3/2's brings you to a C that is sharper than the one you started with, by a ratio of 531441/524288. This ratio is known as the Pythagorean "comma," equal to 23.46 cents. This flaw in the scale is said to have been a closely guarded secret to Pythagoreans, who would punish by drowning any acolyte who dared to reveal it to the uninitiated (although such stories are apocryphal; the same is told about when one of Pythagoras' students discovered that the square root of 2 was irrational—because Pythagoras believed the universe was only based on rational numbers). Nonetheless we have retained the concept of the Circle of Fifths in Western music, and have made the tuning fit by tempering each fifth to 3 cents flat, thereby spreading the comma over the tonal spectrum by fudging the math.

Now that these concepts are fully explained, let us take another look at the Lambdoma with the associated 12-TET tone names added for correspondence with the Partch system.

Fig. 5: Pythagorean Table with Pitch Names based on Partch's G=1/1[21]

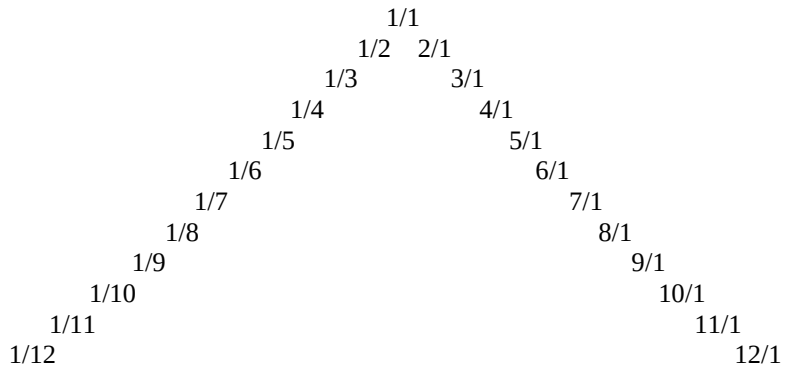


(Drawing & Photo by Author)

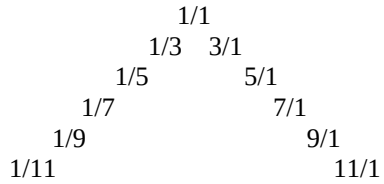
THE TONALITY DIAMOND

In order to make clear how the Lambdoma becomes the Tonality Diamond we will go through a step-by-step process of conversion from one to the other. First we will revert the Lambdoma back to the original Lambda configuration (see Chapter 4, Plato), and remove all the ratios between the outer arms.

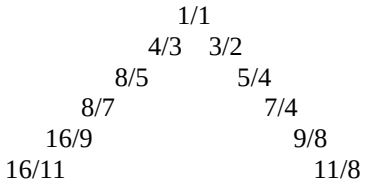
[21] In accordance with what was said previously, the numbers 9/5 and 9/10 should be listed as +↑F, and the numbers 11/5 and 11/10 should be listed as -↓A.



Next we remove any unnecessary numbers by eliminating all octaves. This effectively does away with any powers of 2 or primes multiplied by powers of 2, so ratios containing the numbers 2, 4, 6, 8, 10, and 12 are eliminated. (It may appear that all we are doing is removing the even numbers, but that is not the reason for the removal, and even numbers will reappear later in the process.)



The next step is to modify all ratios other than 1/1 so that they fit within the equation $1/1<x/y<2/1$, thus becoming reduced to the octave between 1/1 and 2/1. This is achieved in each case by changing the number 1, in either the numerator or denominator, into the lowest power of 2 that serves the purpose. In the end, it becomes this:



Next it is necessary for the 3 and 5 identities to change places. This was done by Partch for practicality's sake in making a playable instrument; with those pitches changed

around the lines become a stepwise progression of various types of thirds (12-TET lingo), so long as you are descending in pitch along the left (undertone) arm and ascending the pitch along the right (overtone) arm. If we add the pitch names this should be easy to see.

(G)
1/1
(↑E♭) 8/5 5/4 (↓B)
(C) 4/3 3/2 (D)
(↑A) 8/7 7/4 (↓F)
(F) 16/9 9/8 (A)
(D♭) 16/11 11/8 (C♯)

Each arm now comprises what Partch would call an *otonality* and a *utonality*. In Partchian terminology a *tonality* is a set of six pitches starting at a numerary nexus (in this case, 1/1 is the nexus), and the other pitches relate to the nexus according to the overtone or undertone series in a sequence of 1-5-3-7-9-11—hence, o-tonality or u-tonality, respectively. Then the next step is accomplished by first rotating the lambda ninety degrees to the left, so that 1/1 sits in a far left marginal position, and then adding a 1/1 generator line across the middle.

11/8
9/8
7/4
3/2
5/4
1/1 1/1 1/1 1/1 1/1 1/1
8/5
4/3
8/7
16/9
16/11

Then the other pitches are filled in. There are different ways of approaching this, and it is somewhat more complicated than filling in the Lambdoma pitches. The simplest way to think about it is to create five more ascending otonalities using the udentities as the nexuses; in other words, begin by filling in the pitches along the ascending diagonal going up from 8/5, using the first line as the model. Since you already have 8/5 and 1/1 in

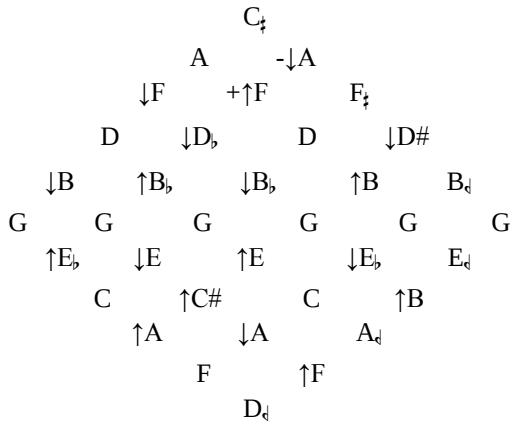
place as the first two pitches of an 8/5 otonality, for the next one you can find the 3 odentity of 8/5 by multiplying 8/5 by 3/2 (adding intervals is accomplished through multiplication). 8/5 x 3/2 = 24/10 = 12/5, then octave-reduced (dividing 12 by 2) to 6/5. The next number would be 8/5 x 7/4 (7/5), the next 8/5 x 9/8 (9/5), and the next 8/5 x 11/8 (11/10). Repeat the same process for each udentity in the lower arm.

It can also be thought of like this: any ratio in the matrix is an intersection of two lines, an ascending diagonal and a descending diagonal. These twelve lines are defined by the prime number in the ratio at the outer left; meaning, for example in the 8/5 ascending otonality line, all denominators must be octaves of 5 (5, 10, 20, 40, etc.). By the same token, any ratio in the 5/4 descending utonality line must have an octave of 5 as a numerator.

The end result will be this:

					11/8						
				9/8		11/10					
			7/4		9/5		11/6				
		3/2		7/5		3/2		11/7			
	5/4		6/5		7/6		9/7		11/9		
1/1		1/1		1/1		1/1		1/1		1/1	
	8/5		5/3		12/7		14/9		18/11		
		4/3		10/7		4/3		14/11			
			8/7		10/9		12/11				
				16/9		20/11					
					16/11						

Hence the Pythagorean Table becomes the Tonality Diamond. For the final basis of comparison, if we look at the Diamond with pitch names in place of the ratios and then refer back to the Table in Fig. 5, you will see that every pitch in the Table is accounted for in the Diamond, without all the repeats involved in the 12 x 12 matrix prior to octave reduction. The only repeats in the Diamond are the 3/2 (D) and 4/3 (C), which each repeat once, and the 1/1 which appears six times across the generator line.



So here the 12 x 12 Lambdoma matrix is compressed from 144 tone-numbers with multiple repeats across eight octaves to a more practical configuration, with 29 pitch-identities spread over 36 positions compressed into a four-octave range. This matrix was the basis of several instruments Partch invented and formed the foundation stone of his musical theory.

In the next chapter we revisit the teachings of Pythagoras via the writings of Plato, in order to gain greater understanding of the philosophical associations involved with the Lambda/Lambdoma.

CHAPTER 4: PLATO

REPUBLIC AND *TIMAEUS*

All scholarship on the Pythagorean Table speaks of it as an elaboration of concepts which appear first in the writings of Plato (whether or not they believe the Table to have truly originated with Pythagoras, I would add). The Myth of Er in Plato's *Republic* gives us a vision of a planetary order based on a musical/mathematical model, but his dialogue *Timaeus* paints a more exact picture. A certain passage originally translated into English by Thomas Taylor (1758-1835) and included as the first article in Godwin's *Harmony of the Spheres* is referred to by the title "The Demiurge Fashions the World-Soul." It is essentially a Platonic creation myth. "The World-Soul carries out several essential, cosmological functions, not the least of which are causing the inerrant motions of the heavens and helping individual souls to sustain and provide for the bodies of the plants and animals that they respectively animate. Plato is emphatic that either the cosmos as a whole—(the World-god)—or, at the very least, the World-soul is a god, and is chief among whatever number of gods (e.g. the stars or just their souls) and demigods (i.e. daemons or guardian spirits) occupy the temporal world and animate the cosmos."[22]

The term *demiurge* comes from the Greek *demiourgos* (δημιουργοζ) and means "craftsman" or "artisan," but it is really speaking of the creative aspect of the One (1/1), or the aspect of the Divine that operates within the physical universe. This concept is also

[22] Jason G. Rheins, "The Intelligible Creator-God and the Intelligent Soul of the Cosmos in Plato's Theology and Metaphysics" [PhD diss., University of Pennsylvania, 2010], 10.

linked to Intellect, an image of the universe as created in the mind of God.[23] The allegory of the Demiurge describes the soul of the universe as having been fashioned according to harmonic laws. Consider this excerpt (spacing, italics and capitalizations are this author's additions).

> From an ESSENCE impartible, and always subsisting according to sameness of being, and from a nature divisible about bodies, he mingled from both a third form of ESSENCE, having a middle subsistence between the two.
>
> And again, between that which is impartible and that which is divisible about bodies, he placed the nature of SAME and DIFFERENT.
>
> And taking these, now they are THREE, he mingled them all into one idea.
>
> But as the nature of DIFFERENT could not without difficulty be mingled in SAME, he *harmonized* them together by employing force in their conjunction.
>
> But after he had mingled these two with ESSENCE, and had produced ONE from the THREE, he again divided this SAME, DIFFERENT, and ESSENCE.
>
> But he began to divide as follows:
>
> In the first place he received ONE part from the whole.
> Then he separated a second part, DOUBLE of the first;
> Afterwards a third, SESQUIALTERS of the second, but TRIPLE of the first;
> Then a fourth, DOUBLE of the SECOND;
> In the next place a fifth, TRIPLE of the THIRD;
> A sixth, OCTUPLE of the FIRST;
> And lastly a seventh, TWENTY-SEVEN times more than the FIRST.
>
> After this, he filled up the DOUBLE and TRIPLE intervals, again cutting off parts from the whole;
>
> And placed them so between the intervals, that there might be two mediums in every interval;
>
> And that one of these might by the same part exceed one of the extremes, and be exceeded by the other;
>
> And that the other part might by an equal number surpass one of the extremes, and by an equal number be surpassed by the other.
>
> But as from hence SESQUIALTER, SESQUITERTIAN, and SESQUIOCTAVE intervals were produced, from those bonds in the first spaces, he filled with a SESQUIOCTAVE interval all the SESQUITERTIAN parts, at the same time leaving a part of each of these.
>
> And then again the interval of this part being assumed, a comparison is from thence obtained in terms of number to number, subsisting between 256 and 243.[24]

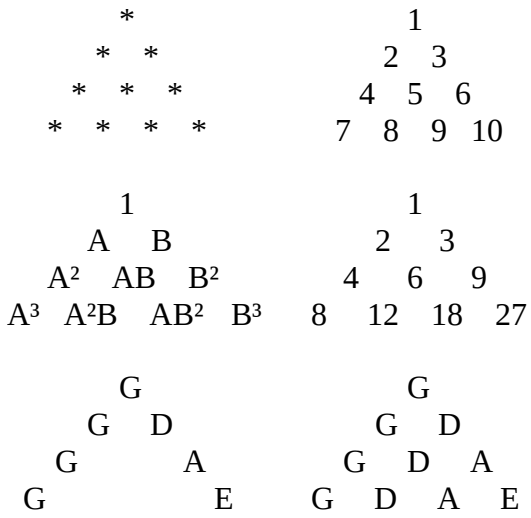
[23] Ibid., 12.
[24] Plato, "The Demiurge Fashions the World-Soul." In Godwin, *The Harmony of the Spheres*, 4.

The crucial passage above begins following the Demiurge's mingling of incompatible substances into a unified trinity—the divisions which describe a figure set out in the form of the Greek letter *lambda* (Λ).

1
2 3
4 9
8 27

Recalling our earlier discussion of the powers of 2 and 3 in just intonation, we can see this model reflects these two fundamental musical laws, the Law of Octaves and the Law of Three. Of course it also a progression from Unity to the first two primes, and then to the square and the cube of each, making it symbolic of the process of manifestation into three-dimensional space. This design is also known as the *tetraktys*, and is considered one of the most sacred Pythagorean symbols/teachings.

Fig. 6: Various Forms of the Tetraktys



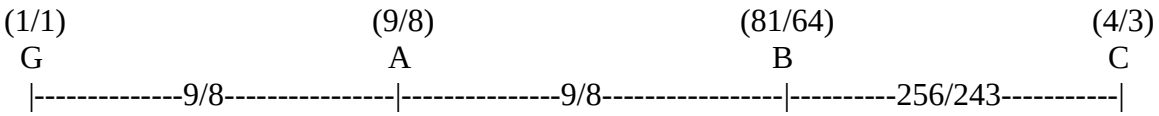
The first Tetraktys used by Pythagoreans is the one with dots instead of numbers, which represents the first four integers and the fact that their sum is 10, considered to be a holy number (1+2+3+4=10). The two on the bottom represent the tonal versions, with G

as the 1/1. The term *lambdoma* is applied to the figures where the coordinates lying between the arms of the lambda are filled in. For more concepts associated with the Tetraktys, see Appendix III.

Moving on through the rest of Plato's allegory, we are given a description of how to arrive at both arithmetic and harmonic *means* between two integers. An arithmetic mean is found with the equation (a+b)/2, while a harmonic mean is established by 2ab/(a+b). If we use two numbers an octave (2/1) apart for our a and b—in this case, 6 and 12—we get a harmonic mean of 8 and an arithmetic mean of 9. The complete version of this numerical relationship can be expressed as 6:8::9:12. If we convert these numbers into tones—temporarily using G as 6—you get G-C-D-G. This configuration contains the three primary consonances, the *diapason* or *proportio dupla* (octave, 2/1, G-G), the *diapente* or *proportio sesquialtera* (fifth, 3/2, G-D & C-G), and the *diatessaron* or *proportio sesquitertia* (fourth, 4/3, G-C & D-G). Thus an octave is said to contain two 4/3's and a whole tone 9/8, or *proportio sesquioctavum*. When looking at the first version of the tetraktys as the holy symbol, comprised of dots, the numbers of the most consonant intervals are there, from bottom to top: 4/3, 3/2, and 2/1. For more on Latin or Greek terminology regarding just intervals, see Appendix II.

The final sentence of the excerpt is a reference to the ratio 256/243, also known as the *leimma*, literally meaning "left over." This refers to the creation of tetrachords by filling in the 4/3 or 3/2 with whole tone steps, which in each case leaves a leftover semitone the size of 256/243. This process is demonstrated below, by filling a 4:3 interval with the tones to make the first tetrachord of a Pythagorean diatonic scale.

Fig. 7: Pythagorean Tetrachord[25]



> Having however mentioned the tetractys, in consequence of the great veneration paid to it by the Pythagoreans, it will be proper to give it more ample discussion, and for this purpose to show from Theo of Smyrna, how many tetractys there are: 'The tetractys,' says he, 'was not only principally honoured by the Pythagoreans, because all symphonies are found to exist within it, but also because it appears to contain the nature of all things.' Hence the following was their oath: 'Not by him who delivered to our soul the tetractys, which contains the fountain and root of everlasting nature.' But by him who delivered the tetractys they mean Pythagoras; for the doctrine concerning it appears to have been his invention. [26]

Therefore in *Timaeus* we see a chain of thought that links number, elementary tone-relationships, planetary motions, the animating forces of nature, and the will of the Divine together within one splendid allegory. This is the essence of *musica universalis*. Yet we must consider, before moving on, the troubling fact of Pythagoras alleged insistence upon 81/64 as the third degree of the diatonic—a rather dissonant and inharmonic interval when compared to the more natural 5/4 found within the octaves of the lower harmonic series, far more accessible and acceptable to the ear. This could be an early example of how the desire to make reality conform to theology overwhelms reasoned observation—the sad fact of which led humanity to separate theology from science in the first place.

THE NICOMACHUS-IAMBLICHUS CONNECTION

In tracing the lineage of the Lambdoma, we find ourselves at somewhat of a disadvantage if we are not able to read German or classical Greek. The later German

[25] Note: the 256/243 semitone is smaller than the just 16/15 semitone by 21.4 cents, and smaller than an equal tempered semitone by 9.8 cents.
[26] Thomas Taylor, *The Theoretic Arithmetic of the Pythagoreans* [York Beach, ME: Samuel Weiser, 1983], 183.

Neopythagoreans who resurrected the Lambdoma speak only of one text which alludes to the concept directly, that being a commentary by Iamblichus regarding the *Lesser Arithmetic* of Nicomachus. Our modern sources who consulted the ancient treatises, Albert von Thimus and Hans Kayser, do not indicate any other written source. Some critics have intimated that these modern scholars invented the concept themselves with the inspiration of the ancients, simply expanding upon the earlier Platonic concepts of the Lambda/Tetraktys. Rather than making any ill-informed judgment regarding this we shall assume good will and take the modern scholars at their word.

In Kayser's masterwork *Lehrbruch der Harmonik* (1950), only available in English for the first time in 2006, he relates the source passage from Iamblichus as such:

> Let us first take unity, and delineate, as from an angle of the same, a figure in the form of the Lambda Λ...and complete one of the sides according to the series with the numbers that follow unity, continuing as long as we like, e.g. 2 3 4 5 6 etc. The other side, though, beginning with the largest of the divisions, which in size is half of the whole lying next to it, will be completed according to the series with the adjacent divisions, 1/3 1/4 1/5 1/6 1/7 etc. Then we will be able to see the aforementioned interplay of two similar things, and the equilibrium of joining…and dividing…which we have just drawn. And while the whole is said to be divided in two halves, it shows, like a double yoke, both the half and the double. It behaves in the same way when a third comes from the division in three, and a fourth from the division in four. And thus it goes on to a hundred, a thousand, ten thousand, showing us in a compelling manner (on the right) how division must continue to the infinitesimal degree, and (on the left) how the same numbers eternally expand to infinity.[27]

Thus the Table is described but not laid out in detail, and it was left to later scholars to expand the concept. A full expansion in print, amazingly, would not occur until 1600 years later, with a German amateur scholar whose work is only known to a few to this day.

[27] Hans Kayser, *Textbook of Harmonics* [Sacred Science Institute, 2006], 119.

CHAPTER 5: THE GERMANIC REVIVAL

ALBERT VON THIMUS

The man credited by all twentieth century scholars with reviving the concept of the Lambdoma was the Prussian politician Albert Freiherr von Thimus (1806-1878). He was primarily a lawyer, judge and member of the German Reichstag, but was also an incredibly learned and dedicated amateur scholar. The scope of his studies included the *I Ching* and the *Tao Te Ching* in the original Chinese, Egyptian hieroglyphics, Greek philosophers, the Jewish *Sefir Yetzirah*, Christian doctrinal tracts, and all works on music theory published since the Middle Ages. He sought to prove that the connection between harmonics and mathematics was the underlying, unifying factor behind all philosophy, theology and mythology the world over. To this end he penned an incredible scholarly work, *Die harmonikale Symbolik des Alterthums* (Harmonic Symbolism in Antiquity). Two volumes of this work were published in 1868 and 1876, with plans for a third that was never completed before his death.[28] This work was largely ignored during von Thimus' lifetime and has yet to be translated into English in its entirety. Only a fragment is accessible to those who cannot read German—a segment on the Pythagorean Table translated by Prof. Joscelyn Godwin and included in his anthology *The Harmony of the Spheres*, a sourcebook of Pythagorean writings dealing specifically with the concept of *musica universalis*.

> The harmonic structure of the upward and downward rows of notes grouped into a single tone-system comes, as the development of the lambdoma-table from the abacus has shown, out of a sequence of major and minor chords together, whose construction according to the numbers of the senary derives its proportions from the multiples of the increasing series and the fractions of the

[28] Joscelyn Godwin, *The Harmony of the Spheres*, 370.

> decreasing ones. The number of instances of the arithmetic and harmonic progressions (resembling the links in a chain, joining them together in a larger series) out of which these major and minor chords arise never exceeds six. There are only three prime numbers, which, in their multiplications and divisions with themselves and with each other, provide the number-forms to whose musical apprehension the intuitive power of our soul—very narrowly circumscribed, here below—is receptive. For beings of a higher sort (and doubtless also for our souls in their erstwhile community with the choirs of holy Spirits called to the contemplation of God) the direct intuitive perception of the laws of symmetrical beauty which find their expression in numbers is certainly not limited by such narrow boundaries. As the sensible expression of the endless proportions of the harmonies concealed within the depths of the very Godhead, the two number-series of increasing and diminishing quantities, and of accelerating and decelerating regular movement, find in the concepts of the infinitely large and the infinitely small not so much their limits as their unitary starting point. Before the Lord God the infinitely large is Nothing and the infinitely small is All: both forms of this infinite duality are One. To Man, created in God's own image, is given, even during his existence in bondage to the sense-world here on earth, the capability at least of mounting upward toward the vision of that Truth through the thinking that releases him from the fetters of corporeality.[29]

To clarify some of the points in the above passage: when von Thimus speaks of "the numbers of the senary," he is referring to the first six numbers along each of the arms of the Lambda (respectfully, 1, 2, 3, 4, 5, 6, and 1, 1/2, 1/3, 1/4, 1/5, 1/6; for a visual refer to Fig. 5), and pointing out that the formulas for both major and minor chords are found within them; thus the foundation of all our Western music is within these first six numbers. Seven, and all prime numbers thereafter, are thought by von Thimus to introduce tones that are not harmonically viable, at least to earthly music (Partch, of course, would disagree). But von Thimus does concede that the music of so-called "higher beings," presumably the angels and devas and enlightened souls, is not confined by such limitations. So, an interesting paradox here, considering the contrast with the work of Partch, who claimed the seventh and eleventh partials as consonances; von Thimus would claim Partch's music to be more for heavenly beings, while Partch was seeking to create a music more connected to the body and earth! (See Chapter 7 for more of Partch's ideas on the concept of musical *corporeality*.)

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[29] Albert von Thimus. "*Die harmonikale Symbolik des Alterthums:* Pythagorean table; Ancient heliocentricity." In Godwin, 371.

HANS KAYSER

Hans Kayser (1891-1964), a German composer/theorist and art historian, studied art and natural science at the University of Berlin and music at the Hochschule für Musik with Engelbert Humperdinck. He later studied composition privately with Arnold Schoenberg, who suggested to him eventually that he was more a philosopher than a composer. He received his doctorate in aesthetics/art history at the University of Erlangen (1916). In 1919 he became General Editor of the series *Der Dom: Bücher Deutscher Mystik*, contributing two of the 13 volumes himself (on Paracelsus and Böhme). One volume was devoted to Johannes Kepler (1571-1630), the German astronomer who authored *Harmonices mundi* (*Harmonies of the World*, 1619). Kepler, while seeking to prove that the velocities of the planets were related according to harmonic laws, discovered one of the fundamental physical laws regarding planetary motion: that the squares of the periodic times are proportional to the cubes of the mean distances. It is possible that working on this volume provided Kayser with his first exposure to the idea of *musica universalis*.[30]

Kayser wrote several books on the "harmonic sciences," including *Vom Klang der Welt* (*The Sound of the World*, 1937), *Abhandlungen zur Ektypik harmonikaler Wertformen* (*Essays on the Ektypics of Harmonic Value-Forms*, 1938), and *Grundriss eines Systems der harmonikalen Wertformen* (*Plan for a System of Harmonic Value-Forms*, 1938); these three books would form the major part of the foundation for *Lehrbuch der Harmonik* (*Textbook of Harmonics*, 1950, English translation published 2006).

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[30] The Hans Kayser Translation Project. "Hans Kayser: a brief Biography," by Ariel Godwin. http://hanskayser.com/EZ/kayser2/kayser2/index.php [accessed January 20, 2013].

In 1964 Kayser published a short book intended as an overview of all the material from the larger volumes above titled *Akróasis: the Theory of World Harmonics*. The following is a passage discussing the initial effect of the Pythagorean tone-number philosophy on those who studied it.

> A bridge between being and value, world and soul, matter and spirit, was found. Only from this viewpoint can we begin to comprehend the tremendous significance of the idea of harmonics in classical antiquity. This spiritual and intellectual side of Pythagorism was cultivated only as esoteric knowledge; therefore it was soon lost and only the other, material side (the reduction of qualities to quantities) remained and was pursued further. Hence the contemporary student of harmonics is not at all surprised by the uncontrolled madness of a natural science and technology which, if not counterbalanced by psychic and ethical values, end by running amok. And for his part, as far as is possible, the modern harmonicist must set himself the task of finding a new approach on the basis of the genuine old "harmonics," one which will do justice even with the present day world view to *both* those Pythagorean approaches and thus revitalize modern thinking in a new way.[31]

Kayser's stated goals involve not just convincing the world of harmonics as the one science that underlies all the rest, but also that knowledge of harmonics would essentially put the soul back into science, reviving a sense of unity with the natural world that would serve as a guiding factor in scientific inquiry and endeavor. His work examines in great detail the correspondences between harmonics and the growth cycles of plants, rocks, and crystals, as well as the principles of harmonic structure at work in painting, sculpture, and architecture. His work is most well known in Germany but is also popular in Switzerland, Austria, Norway, and the Netherlands. In the United States he is relatively unknown, but his work has served to inspire a select few composers, theorists, artists, and philosophers.

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[31] Hans Kayser, *Akróasis: the Theory of World Harmonics* [Boston: Plowshare Press, 1970], 29.

LEVY, LEVARIE, MCCLAIN

Ernst Levy (1895-1981) was a Swiss musicologist, composer, pianist, and conductor. He studied in Basel with Hans Huber and Egon Petri, and in Paris with Pugno. He was the head of the piano master class at Basel Conservatory (1917-21), and later the founder-conductor of the Choeur Philharmonique in Paris (1928), who gave the first performances of Brahms' *German Requiem* and Liszt's *Christus*. He emigrated to the U.S. in 1941 to teach at New England Conservatory (1941-45), Bennington College (1946-51), University of Chicago (1951-54), Massachusetts Institute of Technology (1954-59), and finally Brooklyn College (1959-66), where he met Siegmund Levarie.

Siegmund Levarie (1914-2010), an Austrian musicologist and conductor, studied conducting at New Vienna Conservatory (and graduated in 1935) and musicology with Robert Haas at University of Vienna (received Ph.D. in 1938) and concurrently took private composition lessons with Hugo Kauder. He emigrated to US in 1938 and became naturalized in 1943. He served as director of concerts at University of Chicago (1938-52), conducted the Collegium Musicum, an ensemble specializing mostly in programs of medieval and Renaissance music. He was also Dean of Chicago Musical College (1952-54) and finally Professor of Music and Head of the Music Department at Brooklyn College (1954-62), where he met Ernst Levy, and worked with him on the books mentioned below.

Levy and Levarie worked together at Brooklyn College to develop a unique kind of musical training that would unify music theory with acoustics. They co-authored a book about their program called *Tone: a Study in Musical Acoustics* (1968) which gives credit to Kayser (and von Thimus) for the revival of the Pythagorean Table as "a key to

Pythagorean philosophy and to number symbolism; an explanation of the harmonical method used by Kepler in establishing his astronomic laws; and, in general, the uncovering of a harmonical background in a wealth of phenomena."[32] *On the Sensations of Tone* and Kayser's *Lehrbuch der Harmonik* are both referenced in the work, and no doubt Levy and Levarie could have been exposed to both works in their original language.

> Beyond the immediate musical lessons, acoustics leads us to thoughts that have a bearing on our total life. First, the study of acoustics demonstrates the advantages and virtues of returning to fundamentals. The monochord has *one* string, and the experiments on it represent *one* kind of operation. There is a concentration on the basic phenomenon of a single tone in relation to other tones. The complications that arise are endless; but all music, complex as it has become, is covered by the definition just given: the relation of one tone to other tones…Second, acoustics teaches the need for norms—better still, the recognition that norms exist whether we like them or not. The octave is a norm…The triad is a norm. There are others that emerge strongly and logically as one studies acoustics. Music cannot exist without norms. A sensible course of action is not to fight the norms but to find out how best to operate in relation to them. This is the real meaning of harmony…Third, a universal lesson to be drawn from acoustics is the significance of hierarchy. We mean a hierarchy of values and not of statistics—a hierarchy of qualities (in which good art is rated above less good art) and not merely of quantities (in which eight cylinders are automatically deemed preferable to six)…the ideal interpretation of this concept certainly does not consist in a leveling of values, of taste, and of standards of behavior. Rather, it stems from the recognition, so demonstrable in acoustics, that everything has its definite place in relation to everything else and, by finding it, reaches its full individual and social potential.[33]

Here Levy and Levarie are drawing parallels, in the style of true Pythagoreans, between quantity and quality. Even examining our English words reveals to us a connection: *values* can be defined alternatively as numbers, monetary amounts, intrinsic worth, or moral/cultural priorities. Also, the word *meaning* has a mathematical connotation; and the arithmetic or harmonic *mean* between two numbers represents the point of connection. Levy and Levarie reference the Lambdoma on page 35 of their work, offering more philosophical connections to the number matrix.

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[32] Siegmund Levarie & Ernst Levy, *Tone: a Study in Musical Acoustics* [Kent, OH: Kent State University Press, 1968], viii.
[33] Ibid., ix.

> The consequences of the Pythagorean table for the theory of harmony are numerous and remarkable. Beyond the immediate significance, the table contains philosophic implications that transcend the purely historic interest. Beside having a veritable key to Platonic-Pythagorean philosophy, and Thimus and Kayser have shown, the concepts suggested by the table are not confined to a singular school of thought but have been operative through the Middle Ages and beyond. The Omega concept of the modern scientist and philosopher Teilhard de Chardin, for instance, is easily equated with, and precisely elucidated by, the 0/0 point of the lambdoma. Nor did Christian theologians fail to recognize the symbolic representation by the table of the Holy Trinity at the 0/0 and 1/1 points ("three in one," "two in one") and of the process of creation and procreation inside the expanding network.[34]

> A curious analogy exists between the Pythagorean table and the periodic table of elements. In both cases, a linear series of numbers came to be known. The series was felt to be significant, but its hidden law was not discovered before the application of an "artificial" arrangement by means of interpolation resulting in planimetric development. Both the lambdoma and the periodic table are essentially projections of the structure of our mind, regardless of the possibilities of their application to the outer world. As such, they may be called "icons" in the truest and purest sense: images of forms or our spirit. The periodic table of the chemist has no reality in the ordinary sense, for nowhere are the elements found packed together in the order determined by the table. We deal here with an inner form concept which, first, has an intrinsic value; second, proves to be applicable and thereby demonstrates an attunement between the inner and outer worlds; and, third, shows the existence of a higher kind of order that remains invisible under ordinary circumstances. The same remarks hold true of the Pythagorean table, the differences lying primarily in the material contents—musical tones instead of chemical elements.[35]

The correlation of this linkage between "inner and outer worlds" is also inherent in the Lambdoma itself. Review the table and notice the 1/1 as repeating entity moving diagonally through the center of the matrix (1/1, 2/2, 3/3, 4/4, 5/5, 6/6…). This 1/1 generator line, as it is called, divides the world of numbers from quantities less than one becoming progressively smaller from quantities more than one growing progressively larger, each potentially proceeding all the way to 1/∞ and ∞/1, respectfully. This mathematical reality easily intercepts with philosophical concepts such as the duality of man, the polarization of complementary opposites, the doctrine of yin and yang, etc.

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[34] Ibid., 37. Teilhard de Chardin (1881-1955), a French Jesuit priest, philosopher, paleontologist and biologist, spent his life in a quest to unify science with theology. His concept the Omega Point is described by newworldencyclopedia.org as "the ultimate maximum level of complexity-consciousness, considered by him (de Chardin) the aim towards which consciousness evolves." He believed the process of evolution to be a movement from *biosphere* to *noosphere*, a realm of pure mind in which man and universe are ultimately united. His writings on the subject brought him into frequent conflict with the hierarchy of the Catholic church, who insisted that he contradicted St. Augustine's doctrine of original sin. It is also worth noting that de Chardin spoke often of the evolving spirit of the Earth itself, with striking similarity to the Platonic concept of the World-Soul.

[35] Ibid., 38.

Credit is also given by Levarie and Levy to their colleague Ernest G. McClain (1918-) for the building of the monochord which they used at the school for demonstrative tone-experiments. McClain (Oberlin College B.Mus. 1940, Northwestern University M.Mus. 1946, Columbia University Ed.D. 1959) had worked as the Band Director at Denison University (1946-47) and University of Hawaii (1947-50) before joining the faculty at Brooklyn College, where he taught until he retired in 1981.

McClain, inspired by the Pythagorean investigations of Levy and Levarie, began his own exploration, seeking manifestations of harmonic laws in great works of philosophical literature. His first efforts in this were published as *The Myth of Invariance: the Origin of the Gods, Mathematics and Music from the Ṛg Veda to Plato* (1976). Through the use of "tone-mandalas" and examinations of tuning systems, McClain draws tone-number correspondences as commonalities in various religions and cultures, including ancient Egyptian, Greek, Hebrew, Hindu, Babylonian, and Christian teachings. The primary focus of the work was to examine mathematical allegories in Plato and the Ṛg Veda of India and convert numbers into tones, thus exploring a musical meaning for passages that have confounded analysts for centuries. He followed this up by delving further into Plato in *The Pythagorean Plato: Prelude to the Song Itself* (1978). Both these works are regarded as breakthrough achievements, but highly speculative in their conclusions.

> My work progressed by applying the von Thimus-Levy musical insights to the mathematical foundation provided by Adam, Taylor and Brumbaugh, under the guidance of Siegmund Levarie—mentor, friend, and colleague—who has participated in my studies from the beginning. The result is a thesis which none of us could have anticipated: not only are all of Plato's mathematical allegories capable of musical analysis—one which makes sense out of every step in his arithmetic—but all of his allegories taken together prove to be a unified treatise on the musical scale so that each one throws its light on the others. However, it is perhaps even more remarkable that, when the *Republic*, *Timaeus*, *Critias*, and *Laws* are studied as a group as a unity,

> it then proves possible to explain virtually every Platonic mathematical riddle with help from related passages, that is, in Plato's own words.[36]

Levy's own harmonic treatise based on the Lambdoma, *A Theory of Harmony* (1984), was published posthumously, after being edited by Levarie. Levarie passed away in 2010. According to Joscelyn Godwin, McClain continues his investigations even at age 93 in spite of poor health, seeking to complete a work of harmonic analyses of biblical mathematical allegories.[37] He had previously tackled Islam in *Meditations through the Quran: Tonal Images in an Oral Culture* (1981).

[36] Ernest G. McClain, *The Pythagorean Plato: Prelude to the Song Itself* [York Beach, ME: Nicolas-Hays, 1984], 3.
[37] Godwin communicated this by email to me on 1/22/2013.

CHAPTER 6: THE NEXT GENERATION OF PYTHAGOREANS

RUDOLF HAASE & WERNER SCHULZE

Rudolf Haase (1920-2013) started out studying technology and natural sciences in Berlin until he was drafted in 1943. As a POW in a British camp in Egypt he discovered the writings of Hans Kayser. After his release in 1948 he studied musicology and philosophy at the Universities of Münster, Bonn, and Cologne, and graduated with a doctoral dissertation on Brahms' piano music in 1951. Afterward he went to Bern to study privately with Kayser. He was a Lecturer and the Managing Director at Conservatory of Wuppertal from 1955-62, and a Guest Lecturer at the Institute for Applied Psychology in Zürich from 1962-65. He was granted professorship in 1965 at the Vienna University of Music and the Performing Arts, now one of the largest and most prestigious music colleges in the Western world, training 3000 musicians each year.

Haase embarked on his own personal studies of harmonics in the late 1950's, and continued studying on his own until 1967, when he founded the Hans Kayser Institute for Harmonic Research as a program within the University. Harmonic Research (*Harmonikale Grundlagenforschung*) is the term Haase coined for the "science of harmonics," a study of tone-number relationships that includes investigations into all other arts, sciences, and humanities to see how harmonic laws play their part in these endeavors. For his part he did not shy away from the speculative metaphysics of Kayser's philosophy; in fact, in an article "Harmonics and Sacred Tradition" he penned in 1974, he makes what some might characterize as a rather grandiose claim.

> These epistemological foundations of Harmonics allow some most important inferences. If identical structures have appeared in fields otherwise unrelated to one another (we are thinking above all of the Lambdoma, discussed below), some common plan must obviously have existed;

> for the complexity of these norms excludes chance. The inevitable recognition of this plan necessitates acceptance of an author for it, hence a higher spiritual being. Moreover, the acknowledgement of goals and aims in nature (for this is how harmonic norms appear) brings one to conclude, analogously, that these goals and aims reveal a will that has established them. It already manifests in the microstructures of the cell's nucleus recently discovered by natural science as the "programs" for these goals and aims. Only in this area we are not accustomed to speak in terms of a programmer, but rather for millennia have called this power "God"! In other words, with the help of consistent and firmly based analogical and teleological thinking, harmonics is in a position to produce a morphological proof of God. At this level, of course, one can say no more about God; it must remain open whether God is a person or a principle, whether he is to be considered transcendent or immanent to our world, whether or not the architect of the plan and its executor are one and the same, and other questions. We have not yet reached the end of harmonic metaphysics, however, and a whole series of metaphysical assertions could still be formulated which, at least in part, contain a high degree of cogency, and whose identity with ancient ideas Hans Kayser has shown with many examples.[38]

Haase retired from the University in 1990 (but remained an emeritus), and composer-philosopher Werner Schulze (1952-) took over stewardship of the Institute, renaming it the International Center of Harmonics (*Internationales Harmonik Zentrum*, or IHZ). Schulze got his PhD in Philosophy from the University of Vienna, and is also a bassoonist, author and internationally renowned composer. He has received 8 awards and 5 competition prizes for composition, and his works have been featured in over 550 concerts worldwide. Schulze wrote in 1995 concerning the cross-disciplinary applications of Harmonics:

> At the beginning of the 20th century, *Crystallography* developed as an additional science, through observations of the crystallographer Victor Goldschmidt (1853-1933) in Heidelberg, Germany. He discovered important proportional laws in the structure of crystals, and identified their musical characteristics. But in *Physics* and *Chemistry*, too, there are important laws of proportions, which can be interpreted by Harmonic Research. Max Planck (1858-1947) was convinced that his discoveries in Quantum Theory were analogous to harmonics since only multiples of the Planck's Constant can occur—just as harmonics are multiples of the frequency of a fundamental tone (1:2:3:4:5:6:…). The same laws are also the basis of the Periodic Table of the Elements, in relation to the nuclear charge and the number of electrons…In *Botany* and *Zoology* there are harmonic laws, as well. It is not self-evident that birds' singing has the same basis as human music, since birdsongs are not an imitation of music, but develop independently. However, the majority of laws of proportions can be found in *Anthropology*. Externally, the *human body* is harmoniously proportioned, which has already been recognized in the ancient theory of art, and at present is described by anthropometry. It is also an essential fact that the *physiological rhythms of man* are based on simple proportions which are represented by the numbers 1, 2, 3, 4; heartbeat and breathing, for example, are in a proportion of 4:1. The many rhythms of the human body are

[38] Rudolf Haase, "Harmonics and Sacred Tradition," in *Cosmic Music: Musical Keys to the Interpretation of Reality*, ed. Joscelyn Godwin [Rochester, VT: Inner Traditions, 1989], 92.

> similarly coordinated—especially during sleep at the time of the biological midnight—so that they don't run at random and in confusion. All this has been demonstrated by Gunther Hildebrandt and his institute in Marburg/Lahn, Germany."[39]

Although the IHZ has gradually been marginalized by the University and has seen some decline in enrollment (8-10 students in 2012, 4-5 in 2013), it still remains a viable program under Schulze's guidance.[40] "The University has over 3000 students, but only one or two came this year to my program," Schulze says. "Many students come from other schools, like the University of Technology. But my average students are older professionals, between 40 and 60 years of age. I get mathematicians, architects, painters, chess players, priests. The University of Music has a large music therapy program, and I see many of them. Their regular program teaches them how music therapy works; they come to the IHZ to learn about *why* it works."

A partial listing of lecture topics from the IHZ may include:

> 1. "Pythagorean/Neoplatonist Music Theory from Hellenic Times to the Early Renaissance." Focal points: philosophical theology, definitions and classification of music, math and architecture, with their numerical bases.
> 2. "Unity and Diversity of Music in Ancient Cultures." Emphasis on Asia: China, Indonesia, Philippines, India.
> 3. "Harmonic Fundamentals of Western Music: Meter, Rhythm, Tempo, Numeric Order, Symbolism."
> 4. "Mathematical Fundamentals of our Tone-System: Pure Tonality and Temperament."
> 5. "The Universe at Work: Embedded in the Chrono-Biological Rhythms of Life."
> 6. "Architecture and Landscaping: Cosmos-Energy-Man-Measure."
> 7. "Design: from Furniture to Porcelain Manufacturing."
> 8. "Healing: Integrated Arts: Healing People and Nature."
> 9. "The Living World: Features of a Cross-linked Thinking Beyond the Thresholds of Art, Economics, and Science."
> 10. "Medicine: Applications of Harmonic Therapy within Music Therapy."
> 11. "Natural Science: from Crystallography to Human Biology."
> 12. "State Theory and its Constant Principles."
> 13. "Technology: Motor Construction to Water Research."
> 14. "Zoology: from Birdsong to Bio-Dynamics in the Animal Kingdom."[41]

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[39] Werner Schulze, "Harmonic Thinking." Seni, Journal V/3-4, Institut Seni Indonesia. Jogjakarta 1997, 149-159.

[40] Information and quotes in this section are from a telephone interview I conducted with Werner Schulze on March 16, 2013, as well as from a series of emails throughout that month.

[41] Internationales Harmonik Zentrum (IHZ). "HARMONIK – Lehrgang / Wahlfach / Doktoratstudium." http://www.harmonik.at/studium/ [accessed 1/23/2013; translated by Krista Spradlin]

Schulze has worked to promote "harmonic thinking" even outside the University setting, lecturing and presenting at World Expos in Spain (2008), Germany (2000), Switzerland and South Africa (1984), and others. A World Expo gives the opportunity to present to as many as two million visitors, but Schulze is not happy with this sort of promotion; he much prefers the ongoing work of one of his collaborative projects, the Pankratium Gmünd ("The House of Wonder"), a center in Austria he founded with fellow musician Manfred Tischitz and architect Eva Rubin. It is a museum completely devoted to harmonics and the intersection between art and science, and sees over 15,000 visitors a year.

Schulze served as guest professor at the Institut Seni Indonesia in Jakarta in 1995, and has maintained a relationship with the schools there to this day. After his retirement from the Vienna University in 2014 he will go on teaching in Indonesia several times a year. The future of the IHZ in Vienna is uncertain. "There are some candidates for new head of IHZ, but I am a 'colorful dog,'" says Schulze, with a laugh. "Music, philosophy, architecture, biology, healing…I will not be easy to replace." Yet Harmonic Research is by its very nature interdisciplinary, and tends to attract folks with multiple areas of specialty—as well as such seekers who are looking for the thread of commonality in human life that harmonics appears to be.

CHAPTER 7: THE INSTRUMENT BUILDERS

BARBARA HERO & THE PLHK

This author had the opportunity to meet Barbara Hero, a painter-musician-mathematician and probably one of the world's leading experts on the Lambdoma, in person at her home in Topsham, Maine. Born in 1925, Hero began musical training at age 3 under the tutelage of her artist mother, but stopped lessons at age 12 to focus more on visual arts. She pursued education in art at Bennington College and George Washington University (where she received her B.A.), but remained passionate about music, and took classes at New England Conservatory in piano, theory, and composition. Her path as an artist was a search for a unifying theory that would connect art with music. In 1968, while studying Manly Hall's *Secret Teachings of All Ages* for information about Pythagoras teachings having to do with tone-color correspondences, she happened to be perusing the shelves at Boston Public Library—and, by her description, "my hand moved on its own up to a high shelf and took hold of a book, which turned out to be *Tone: a Study of Musical Acoustics* by Levy and Levarie."

Discovering the Lambdoma and its relationship with tone turned out to be a life changer. "It was like a light going off,"[42] she said. The Lambdoma became a full-time focus, with a passion bordering on obsession—to the point that her husband at the time began to believe her mentally unbalanced and divorced her. Still, she pressed on with her painting now Lambdoma-inspired, incorporating the number-tone-color correspondences after deciding upon a reference octave based on middle C at 256 Hz (which is somewhat

[42] The quotes in this section are from interviews I conducted with Barbara Hero at her home in Topsham, Maine, in March of 2013.

lower than the standard 261.63 Hz of the 12-TET system tuned to A440; it is actually a reversion to an older standard of A432, which makes middle C 256 Hz and technically an octave of 1 Hz). She went back to school to earn a Master's Degree in Mathematics Education (1981), and then on to Massachusetts Institute of Technology to study music synthesis, seeking to find an algorithm to enable her to create computer music based on the Lambdoma frequencies.

"After MIT, I began to realize what was happening," Hero says. "I didn't notice it while I was there. Some people would walk through the lab and hear the matrix, and go right down into a lotus position and meditate. Others would go through and put their hands over their ears. So I knew something was going on…" The more she worked with the Lambdoma frequencies, the more she began to conceive of a therapeutic application, which she pursued in earnest after designing the Pythagorean Lambdoma Harmonic Keyboard with Robert Foulkrod, an electronics engineer and her life partner of over thirty years. (Foulkrod—an Army veteran, researcher and designer of new technology systems, and later in life a spiritual counselor—passed away in 2011, and was posthumously honored by Congressman Dennis Kucinich in front of the Ohio House of Representatives.)

The Lambdoma Keyboard (PLHK) is a console of electronic touch-pads laid out in a 16x16 diamond configuration, with the 1/1 generator line going straight up through the center, with each pad color-coded and containing the ratio, 12-TET pitch name, and frequency. The console hooks into a computer and connects with a software program called Fractal Tune Smithy. Hero further divides the console into four quadrants, labeled as such: Quadrant 1—Emotional, 2—Spiritual, 3—Physical, and 4—Oracle.

"I started developing a protocol. First they find a keynote—one pitch on the keyboard they like the best. Then the keynote they pick becomes the 1/1, the generator pitch for the entire matrix. After that is set they go to each quadrant and find a chord that sounds pleasing to them. Then I have them attach a word to each chord, whatever word they automatically associate with the sound. After that I ask them to formulate a question they want answered, and then to compose sentences to go with each chord-word. So that's what I am doing now; first the keynote, then the words, then the question, then the sentence for each word. And almost every time, within the sentences they write is the answer to the question they posed," Hero adds. "I don't charge money for these sessions. People come from all over for them. I just feel this is really important for people to experience."[43]

Besides the PLHK, Hero has used the Lambdoma to create paintings, necklaces, mandalas, and candles. She has recorded a number of CDs using the PLHK, including *The Healing Nature of Sound*, *Grand Gallery Galaxy Sounds*, *Chakra Attunement Chords*, *Fibonacci Chords*, *Lambdoma Chakra Meditations*, and more. She has written several books, including *Ears + Eyes = Ideas*, *Lambdoma Unveiled (The Theory of Relationships)*, *The Glass Bead and Knot Theory of Relationships*, *The Lambdoma Resonant Harmonic Scale (P, Q, R, S, T, U, V, and W)*, and a manual for building your own PLHK.

[43] Author's Note: I am particularly grateful for having had the opportunity to meet Barbara in person during the writing of this paper, and to have her do a session for me with the Lambdoma Keyboard. I can only say that the PLHK generated an internal experience for me that was probably inexplicable by any scholarly or scientific means, the kind that might be perhaps reminiscent of what those who knew Pythagoras himself might have experienced at his hands, accompanied by lyre. Having devoted my adult life to the study of sacred musical instruments used as part of healing rituals in cultures around the world, I confess myself surprised at having such a response to an electronic instrument.—S.E.

She has also designed an instrument called the Tetra-Harp, a 36-string harp built in a tetrahedron shape, with the help of Tim Gordon Anderson, a musician/instrument builder from Edina, Minnesota. She has lectured about her work all over the world (including Bulgaria, Lithuania, England, and China), at various conferences regarding music, psychotronics, engineering, Pavlov, and holistic health. She has written peer reviewed articles for Art & Technology, Mathematics, Pavlovian, and Music Journals since 1975. She founded the International Lambdoma Research Institute in 1997. Her products are available online through her website www.lambdoma.com.

HARRY PARTCH & THE DIAMOND MARIMBA

Early on in his treatise, *Genesis of a Music*, before Partch enters into any discussion of theory and ratios, he talks about his notion of *corporeality* in music, mainly as a stylistic ideal. "Corporeal music is emotionally 'tactile,'" he writes. "It does not grow from the root of 'pure form.' It cannot be characterized as mental or spiritual. The word Abstract, on the other hand, may be used to denote a mass expression, in its highest application, the spirits of all united into one and transported into a realm of unreality, neither here nor now, but transcending both."[44] He drew this distinction in order to fully characterize his own artistic goal of realizing a unity of drama, dance, spoken word, and music; he wanted to produce an artistic experience that was as much felt in the body as it was appreciated by the mind.

This can be seen as somewhat of an Anti-Platonic stance. Partch clearly resented the Platonic concept of *form* as being a higher level of reality. Plato's concept of the realm of perfected ideas (the capitalized *Forms* to indicate a hierarchy above the physical

[44] Harry Partch, *Genesis of a Music* [New York: Da Capo Press, 1974], 8.

plane) being vastly superior to imperfect material manifestations, a notion that has certainly impacted Judeo-Christian theology with its own concepts of "high" heaven and "lowly" or "fallen" earth, would have been offensive to Partch, if the words of his musical diary *Bitter Music* are any indication.

> *Have you confessed your sins to Jesus?*
> My dear boy, get this:
> The sin of your body was invented by a gang of holy quacks.
> If you like their invention, that's your bizness. I don't.
> The practice of the common virtues of honesty, kindness,
> Understanding, consideration for mankind,
> Has nothing to do with religion and never has had anything to do with it.
> And religion as it is practiced is one of the kuh-ur-ses of the race.[45]

Or, elsewhere:

> Last night I laid my bag on the dry leaves under a pepper tree in Ojai Valley. The berries are red and dry now, and sweet to taste.
>
> Tonight I am camping under a live oak tree with a heavy roof of leaves.
>
> I take off all my clothes, as I always do before crawling in, and gaze down at a body pale in the blackness. It is beautiful.
>
> My hands stroke its belly, and I am very happy this November night looking up into the inky O-high oak…
>
> *…I don't want your body. Jesus doesn't want it…*
>
> But I do, and I think it is beautiful.[46]

Let us recall the dichotomy between von Thimus' and Partch's views regarding music incorporating prime number partials of seven or higher (refer to pg. 38-39). The former would characterize any such music as too "heavenly" for the human ear to handle, while Partch embraces them according to the ideal of corporeality, music of a tuning that reflects the harmonious proportions of earthly matter. It is certainly arguable that a preference for just intonation over equal temperament can be seen as part of this argument for corporeality over abstraction. Microtonalists would say that nature itself has already shown us how intervals are meant to be tuned, and so to knock them all out of

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[45] Partch, *Bitter Music: Collected Journals, Essays, Introductions, and Librettos* [Chicago: University of Illinois Press, 2000], 124.
[46] Ibid., 108.

tune as part of some artificial ideal of conformity—the tyranny of the piano keyboard—is a crime against nature, and music, and therefore humanity itself. When one views the historical movement toward 12-TET against the backdrop of the rise of the industrial state and the gradual movement away from any sense of reverence and/or responsibility for the planet, the metaphorical possibilities abound!

Although Partch most likely would not have shared many of von Thimus' views, and certainly Partch shows a disdainful attitude toward religion in his writings, this does not necessarily preclude belief in a magical/supernatural aspect of music-making that many microtonalists claim to be tapping into. Indeed, if there is any kind of spirituality in Partch's work, it would be closer to the polytheism of ancient Greece or the earth-based animism of the Yaqui Indians he knew in his youth. Pythagorean teachings have been embraced in the modern age by those who eschew religion in favor of mysticism and esoteric study, such as the Rosicrucians or Sufis. Consider:

> Microtonal tuning has a long-established connection with occultism and magick. The occult doctrine of correspondences has its roots in the protoscientific philosophies of medieval Europe, with its deep roots in Arabic, Byzantine Greek and older confluent cultures. It had its greatest resurgence during the 15th century Renaissance Neoplatonic revival led by Marsilio Ficino, and its essentials were codified by Cornelius Agrippa in the 16th century. Epitomized by the "as above, so below" axiom of the Emerald Tablet of Hermes Trismegistus, the doctrine posits a one-to-one correspondence between the celestial and earthly realms, and to its initiates, between the inner and outer worlds. The elements, humors, planets and zodiacal signs were understood to cross the macrocosm-microcosm boundary, with counterparts in the human body and spirit. Religious theurgists such as Ficino employed a variety of means, but especially music, to harness these magical correspondences in the quest for health, happiness, and a deeper understanding of God and nature.[47]

> Communication, if it functions at all, comes in many disguises: in plain words, in artfully inflected words, or perhaps no words at all; perhaps telepathically or, according to some, as the result of trans-migratory souls recognizing each other from former lives. In any case, there *is* such a thing as extraverbal magic. And extraverbal is something I now wish to invoke.[48]

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[47] William Breeze, "De Harmonia Mundi." In Zorn, *Arcana V: music, magic and mysticism* [New York: Hips Road, 2010], 11.

[48] Harry Partch, "A Soul Tormented by Contemporary Music Looks for a Humanizing Alchemy: *The Bewitched*." In *Bitter Music*, 239.

The above quote is from a pre-show speech Partch made in 1957 before a performance of his dance-satire *The Bewitched*, a music theatre piece characteristic of the works he created later in life as a final manifestation of his corporeal philosophy. In these works he sought to unite speech, dance, and music as a unified whole (*Delusion of the Fury, Revelation in the Courthouse Park, Oedipus*) with musicians onstage with his instruments and even participating in dramatic or movement elements of the play. Though Partch's works suffered somewhat from his inexperience with theatre and his need for absolute control over all aspects of the productions, he still managed to create compelling work, and with intentions that could be described as incorporating a magic or transformative element.

> Building on the idea of a whole-experience reaction, Partch compared his approach to what he imagined a prehistoric approach might have been; one that made use of "magical sounds," "visual form," and "experience-ritual." In his explanation he suggested that prehistoric cultures brought visual and sonic forms together in order to create a ritualized and spiritual experience. The ritualistic quality of Partch's later works were designed to affect "communal discourse", and that, like ancient Greek drama and Japanese Kabuki, his works had "social-religious" ends. According to Bob Gilmore, the complex elements Partch's theatrical works were intended to disorient audiences for the purpose of preparing them for an experience "parallel to that undergone in ritual."[49]

In spite of Partch's rancor toward religious hypocrisy, he did indeed believe in the transformative power of art—the ability of the performing arts to facilitate shifts in consciousness for people. He envisioned a music theatre experience that recaptured an "ancient magic" that had been lost; and this was not only on the larger macro level of the created piece, but on the micro level with the tools of expression—the tonal system and the instrumental sounds, both of which would confound an audience with harmonies and sonorities completely unfamiliar and seemingly from a more ancient time. This was never with the intention of creating dogma or establishing a new systemic precedent, of course;

[49] Brian Timothy Harlan, "One Voice: a Reconciliation of Harry Partch's Disparate Theories." PhD diss., University of Southern California, 2007.

Partch was forever the rugged individualist and saw his methods as being suitable for himself and the music he was driven to compose—not as something *everyone* should be doing.

It is even more incredible that Partch inadvertently took the concept of the Lambdoma and translated it into the Tonality Diamond, and then brought it into materiality as the Diamond Marimba, thus transmuting the abstract theory into corporeality. Partch built the instrument in 1946 in Madison, Wisconsin, with the help of Warren E. Gibson. The blocks are a mixture of Pernambuco and Brazilian rosewood, and the resonators are Brazilian bamboo.[50] This instrument uses the precise configuration of the Tonality Diamond for its layout of pitches, and although Partch uses the 29 pitches of the 11-limit system as the basis for several other instruments he built (Kithara I & II, Bass Marimba, the core of the Quadrangularis Reversum), the Diamond Marimba is the only one that serves as an exact physical manifestation.

As Partch continued to work with the Tonality Diamond he made the decision to fill in some large intervallic gaps that occur in the 29-tone 11-limit scale with some extra pitches, eventually settling on a 43-tone scale as the basis for his system and instruments (although his compositions are not limited to those pitches; see Appendix V for a complete explanation of the 43-tone scale). Choices he made in this regard reveal the depth of his research and understanding of Pythagorean music theory. For instance, the first pitch above 1/1 in his system is 81/80. The interval 81/80 is known as the comma of Didymus[51] (also called the *syntonic comma, diatonic comma,* or *Ptolemaic comma*). It is

[50] Partch, *Genesis,* 262.
[51] A Roman musician/theorist who lived somewhere around the time of Christ.

the interval of difference between a just major third (5/4) and a Pythagorean *ditone* (81/64).[52]

Yet regardless of how much research Partch did for *Genesis of a Music*, the fact remains that it would have been virtually impossible for Partch to have known about the Lambdoma before writing it. He did not speak German, Greek or Latin, and the first English mention of the Lambdoma in print would have been Levy and Levarie's *Tone*, published in 1968. So how did this seemingly uneducated man, who lived as a hobo for the better part of twelve years, manage the spectacular accomplishment of advancing Pythagorean music theory forward even beyond where von Helmholtz or von Thimus trod? One wonders what Rudolf Haase might have thought about it, given his writings about the likelihood of such things being chance occurrences.[53] Even more intriguing is the fact that both Hero and Partch, the only ones to invent Lambdoma instruments (for the Diamond Marimba certainly is a Lambdoma instrument in spite of Partch's supposed ignorance of the original Pythagorean concept), both did so with what could be deemed "magical" intention—one to heal, and one to forcibly alter consciousness and perception.

[52] It also appears throughout Partch's system as the interval of difference between 10/9 and 9/8, 32/27 and 6/5, 4/3 and 27/20, 40/27 and 3/2, 5/3 and 27/16, 16/9 and 9/5, and the 43-tone scale ends with the reciprocal tone of 160/81.

[53] Refer back to page 47.

CONCLUSION

This study has aimed to open some profound questions regarding the relevancy and efficacy of our conventional wisdom regarding the philosophers of antiquity and the widely held theories of musical harmony, by way of compiling the history of an obscure branch of ancient knowledge that has reemerged after a long sleep. We call it Pythagorean, in spite of uncertainty as to its true origins, but the name serves to classify it as belonging to a larger framework of endeavor and lineage of great minds. The Lambdoma is many things to many people—a simple mathematical construct, a source for the theory of music, a grand expression of universal principle, the basis for a theology of numbers, a cosmic truth for ongoing contemplation.

To the original question that began our discussion—of whether Plato can be fully understood without knowledge of harmonics—I would say the answer is found with simple logic. Since much of what Plato taught supported the concept of *musica universalis* as the basis for understanding the world, and since he went so far as to declare in *Republic* that music and gymnastics are the only important subjects to train children in until they reach adulthood[54], it would follow that the metaphors involving numbers in *Timaeus*—which are easily explained through harmonics and remain quite a puzzle otherwise—are certainly intended by Plato to be understood through realization of the numbers as tones. They are certainly not intended to be dismissed as nothing more than simple quantities. Plato's metaphors demand an *experience* of these numbers for their greater *meaning* to have impact on the mind of the reader. Without that meaning, numbers are just numbers. To have the musical experience of these metaphors, one must

[54] Jowett, B., trans. *Plato's Republic* [New York: Random House, 1966], 105-110.

understand harmonics. Therefore Platonic scholarship can hardly be complete without that understanding.

To the greater question of whether music theory can be complete without understanding *musica universalis* and the greater significance of harmonics, we are certainly more in the field of this author's area of expertise, and therefore the question begs a longer answer.

There is no doubt that musicians can be good at their craft without even the smallest amount of musical education; some of the most impressive musician/composers of the twentieth century were largely self-taught—Harry Partch being one of them. But many of these same composers, even if they are vehemently anti-religion, claim a sense of music connecting them to the cosmos in a manner that could be characterized as mystical or spiritual—even if they had only heard of Pythagoras in their high school geometry class. Partch may have had no use for religion but he did speak frequently of the idea of music as *magic* in the truest sense—and all sages, shamans, magi, wizards, etc. throughout all history and cultures would wholeheartedly agree.

For Pythagoras and those that followed him, the discovery of the tone-number relationship was precisely what put the magic nature of music beyond the shadow of doubt. Not only could one perceive that magic with the senses, but the structure of reality was rendered knowable through the study of the monochord. The mind of man could unify with the mind of the Creator. We were no longer adrift in the vagueness of intuition; tuning could be calculated to mirror the perfect numbers of the divine construction.

From this majestic starting point, Western music moved through centuries of change whereby its roots were forgotten, and means of expression shifted—from modalism to major-minor tonality to the harmonic experimentation of the twentieth century. Another great composer, the aforementioned Arnold Schoenberg (p. 40), used the concept of the twelve by twelve matrix to do away with all fixed rules of tonality and "emancipate the dissonance," creating a style of composition that sought to end the reign of the 1/1 as the point from which all harmonic relationship emanated. The conventional rules had become synonymous with tyranny and oppression, and music could only be redefined for the post-WWII world by shattering all our preconceptions about its nature.

Of course one cannot shatter what is absolute and unchanging, and the mistake in this case was to confuse the arbitrary rules decided on by unenlightened men (who sought to make music conform to systems of belief) with the laws of the universe which only mathematics could reveal. This is the place to which we can return, if we so choose. What can we wrest from the Lambdoma, using our capacity for poetry and metaphor to tease meaning from these numbers that Pythagoras revered as gods? Let us pause once more to contemplate it.

1/1	2/1	3/1	4/1	5/1	6/1	7/1	8/1	9/1	10/1	11/1	12/1
1/2	2/2	3/2	4/2	5/2	6/2	7/2	8/2	9/2	10/2	11/2	12/2
1/3	2/3	3/3	4/3	5/3	6/3	7/3	8/3	9/3	10/3	11/3	12/3
1/4	2/4	3/4	4/4	5/4	6/4	7/4	8/4	9/4	10/4	11/4	12/4
1/5	2/5	3/5	4/5	5/5	6/5	7/5	8/5	9/5	10/5	11/5	12/5
1/6	2/6	3/6	4/6	5/6	6/6	7/6	8/6	9/6	10/6	11/6	12/6
1/7	2/7	3/7	4/7	5/7	6/7	7/7	8/7	9/7	10/7	11/7	12/7
1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8	9/8	10/8	11/8	12/8
1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	9/9	10/9	11/9	12/9
1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	10/10	11/10	12/10
1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11	11/11	12/11
1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	12/12

The Lambdoma can be extended to include *every conceivable rational number*, and yet the presence of the 1/1 will be eternally present down through the middle all the way to the final corner. If the ratios are given to represent all things that exist, the neutral ground of the 1/1 generator line divides all creation into numbers either greater or lesser than the 1/1, revealing the duality inherent in the world between positive and negative, masculine and feminine, ascending and descending, and all other aspects of reality eternally subject to the laws of polarity.

It is also worth noting that each tone-number-identity displayed here in its aspect as a ratio is a *relationship*, a bridge between two discrete archetypes. What does this tell us? What does it say to define Unity not as simply the One, but as 1/1, the relationship of Unity to Itself? Perhaps it calls to mind the dictum of the Oracle at Delphi—"Know Thyself"—that was once held as the only true path to understanding the universe and our role within it? Perhaps it tells us that relationship is the basis of all harmony, and that tones only acquire meaning by their relationship to the 1/1, and otherwise will only exist as arbitrary frequencies without greater purpose?

However acceptable or unacceptable these philosophical speculations may be to the reader, the fact remains that the Lambdoma gives us these questions to ponder, opening our minds to greater possibilities, and that is the "heart of philosophy" referred to by Jacob Needleman (see page 7). This author recalls that Werner Schulze, when asked the central question of whether music theory is truly complete without knowledge of *musica universalis*, replied with only one word: an emphatic "NO." This author wholeheartedly agrees.

APPENDIX I:

THE THEOLOGY OF NUMBERS

NUMBERS	CORRESPONDENCES
Monad (1)	Instrument of Truth, Obscure, Not-Many, A Chariot, Male-Female, Immutable Truth, Invulnerable Destiny, A Seed, Demiurge, True Happiness, Zeus, Life, God, Equality in Increase and Decrease, Memory, A Ship, Essence, The Innkeeper ("that which takes in all"), The Pattern or Model, The Moulder, Prometheus, The First, Darkness, Blending, Commixture, Harmony, Order, Materia, A Friend, Infinite Expanse, Space-Producer, Intellect, Infinity, Chaos, Chasm, Styx, Rigid Virgin, Atlas, Apollo, Form, Proteus, Mnemosyne.
Duad (2)	Inequality, Indefinite, The Unlimited, Without Form or Figure, Growth, Birth, Judgment, Appearance, Anguish, The Each of Two, Falling Short, Defect, Erato, Equal, Isis, Movement, The Ratio (*logos*) in Proportion, Revolution, Distance, Impulse, Excess, The Thing with Another, Rhea ("flow"), Selene, Combination, That Which is to be Endured, Misery, Distress, Boldness, Audacity, Matter, Obstinacy, Nature, Cause of Dissimilitude, Interval Between Monad and Multitude, Harmony, Patience, Root, Phrygia, Eleusinia, Diana and Cupid, Venus, Ignorance, Falsehood, Indistinction, Strife, Dissension, Fate, Death.
Triad (3)	Proportion, Harmonia, Marriage, Knowledge, Peace, Every Thing, Hecate, Good Counsel, Piety, The Mean Between Two Extremes, Oneness of Mind, The All, Perfection, Friendship, Purpose, First Odd Number, Intelligence, Virtue, Polymnia, Thetis, Ophion, Pluto, Metis, Triton, Mistress of Music.
Tetrad (4)	Nature of Change, Righteousness, Hercules, Holding the Key of Nature, The Greatest Miracle, Aeolus, Mercury, Vulcan, Bacchus, Urania, First Square, Tetractys
Pentad (5)	Alteration, Immortal, Androgyny, Lack of Strife, Aphrodite, Boubastia, Wedding, Marriage, Double, Manifesting Justice, Justice, Demigod, Nemesis, Pallas, Five-Fold, Forethought, Light
Hexad (6)	Resembling Justice, The Thunder-Stone, Amphitrite (Poseidon's wife, "on both sides three"), Male-Female, Marriage, Finest of All, In Two Measures, Form of Forms, Peace, Far-Shooting (Apollo), Thaleia, Kosmos, Possessing Wholeness, Cure-All, Perfection, Three-Fold, Health, Reconciling
Heptad (7)	The Forager (epithet of Athena), Athena, Citadel, Reaper, Hard to Subdue Defense, Due Measure, Virgin, Revered Seven, Bringing to Completion, Fortune, Fate, Preserving

Octad (8)	Untimely Born, Steadfast, Seat, Abode, Euterpe, Cadmia, Mother, All Harmonious
Ennead (9)	Brother and Consort of Zeus, Helios, Absence of Strife, Far-Working (epithet of Apollo), Hera, Hephaestus, Maiden, Of the Kouretes, Assimilation, Oneness of Mind, Horizon, Crossing, Passage, Prometheus, Consort and Brother, Perfection, Bringing to Perfection, Terpsichore, Hyperion, Oceanus
Decad (10)	Eternity, Untiring, Necessity, Atlas, Fate, Helios, God, Key-Holding, Kosmos, Strength, Memory, Ourania, Heaven, All, All Perfect, Faith, Phanes

(Sources: Guthrie/Fideler, *The Pythagorean Sourcebook and Library*, taken from *Theology of Numbers* by Iamblichus, and Taylor, *The Theoretic Arithmetic of the Pythagoreans*)

APPENDIX II:

GLOSSARY OF PYTHAGOREAN MUSICAL TERMINOLOGY

Apotome. 2187/2048, or the tone at 7 3/2's above the 1/1; a form of semitone. Pythagorean semitone.

Arithmetic Mean. A number located arithmetically between two other numbers, found by adding the two other numbers together and then dividing by 2. The arithmetic mean of a 2:1 is at 3:2.

Artios. The dimishing series on the Lambdoma: 1/1, 1/2, 1/3, 1/4….

Diapason cum Diapente. A 3/1, or a 2/1 plus a 3/2.

Diapason Formalis. The formal 2/1. The distinction here between "formalis" and "materialis" has to do with the Platonic concept of the idea of form as the perfected ideal of a thing, versus the imperfected material manifestation of said ideal.

Diapason Materialis. The material 2/1.

Diapente Formalis. The formal 3/2.

Diapente Materialis. The material 3/2.

Diatessaron Formalis. The formal 4/3.

Diatessaron Materialis. The material 4/3.

Diazeuxis. "Disjunction," the whole tone between the tetrachords.

Diesis. 128/125 sharp. This is the interval of difference between a 2/1 and three 5/4's. This term has multiple meanings, however.

Diezeugmenon. Tetrachord of the disjunct notes.

Dioxeian. One name for a 3/2.

Disdiapason. The rich/wealthy 2/1. In actuality refers to a 4/1.

Ditone. 81/64, a Pythagorean major third, the product of 9/8 x 9/8, the combining of two 9/8 whole tones.

Dorian Mode. Originally E-E, now D-D.

Epitritic Proportion. 4/3.

Epogdoan. 9/8.

Harmonia. One name for a 2/1, but not just the interval itself; all that is within it—which is everything, every conceivable interval.

Harmoniai. "Tunings," "attunements"—modes.

Harmonic Mean. The mean which is superior and inferior to the extremes by the same fraction. To find harmonic mean *b* in the sequence *a b c*, apply the operation $b = (2ac)/(a+c)$. The harmonic mean of a 2/1 is at 4/3.

Hemiolic Proportion. 3/2.

Hypate. "Highest." Refers to the tone of highest pitch (in lyre tuning) and the planet occupying the highest space in the heavens. According to Nichomachus, E/Kronos/Saturn. According to Plato, it works in reverse, and the tone is the same but represents the Moon.

Hypaton. Descriptive term attached to a note or tetrachord, placing it below the meson.

Hyperbolaion. Descriptive term attached to a note or tetrachord, placing it in the highest tetrachordal position.

Hypodorian Mode. A-A. Now called Aeolian.

Hypolydian Mode. F-F. Now called Lydian.

Hypophrygian Mode. G-G. Now called Mixolydian.

Leimma. 256/243, or the size of the remaining semitone after inserting two 9/8 whole tones into a 4/3 tetrachord. The name literally means "left over."

Lichanos. Also called **Hypermese**. "Above the middle." Nichomachus puts it at G/Ares/Mars, Plato at G/Aphrodite/Venus.

Lydian Mode. Originally C-C (now called Ionian), now F-F.

Mese. "Middle." Note A. Nichomachus: Sun. Plato: Hermes/Mercury.

Meson. Tetrachord of the middle notes.

Mixolydian Mode. Originally B-B (now called Locrian), now G-G.

Nete. "Lowest." Nichomachus: D/Moon. Plato: E/Fixed Stars/Zodiac.

Paramese. "Next to the middle." Nichomachus: B flat/Hermes/Mercury. Plato: B natural/Ares/Mars.

Paranete. "Next to the lowest." Nichomachus: C/Aphrodite/Venus. Plato: D/Kronos/Saturn.

Parhypate. "Next to the highest." Note F. Nichomachus: Zeus/Jupiter. Plato: Sun.

Perissos. Increasing series on the Lambdoma: 1/1, 2/1, 3/1, 4/1…

Proportio Dupla. The octave or 2/1, the doubling.

Proportio Quadrupla. The 4/1, or double octave.

Proportio Sesquialtera. The "fifth" or 3/2, the interval of difference.

Proportio Sesquinona. The minor whole tone, or 10/9.

Proportio Sesquioctavum. The major whole tone, or 9/8.

Proportio Sesquiquartum. The Just major third, or 5/4.

Proportio Sesquiquintum. The Just minor third, or 6/5.

Proportio Sesquitertia. The "fourth" or 4/3.

Proportio Tripla. The octave plus fifth, or 3/1.

Proslambanomenos. "Taken in addition." A note added at an octave below the mese.

Ptolemaic Sequence. Ptolemy's version of the major scale. 1/1—9/8—5/4—4/3—3/2—5/3—15/8—2/1. 5-limit sequence, as opposed to Pythagoras' 3-limit.

Pythagorean Comma. 531,441/524,288. 23.46 cents.

Pythagorean Diatonic Scale. 1/1—9/8—81/64—4/3—3/2—27/16—243/128—2/1.

Schisma. Half of a comma.

Semiditone. 32/27. The Pythagorean minor third.

Superparticular Ratio. A ratio of two numbers with a difference of 1 between them, such as 9/8 or 3/2 or 10/9.

Syllaba. One name for a 4/3.

Synemmenon. Tetrachord of the conjunct notes.

Syntonic Comma. 81/80, also known as the comma of Didymus.

Tetrachord. A sequence of four notes. A mode/scale is constructed as two tetrachords, the lower beginning at the 1/1 and ending at the 4/3, the higher beginning at the 3/2 and ending at the 2/1.

Tetraktys. 6:8::9:12. In musical terms, a starting pitch and the 4/3, 3/2, and 2/1 above.

Trite. "Third." Same as the **Paranese** in Nichomachus' system. Plato: C/Zeus/Jupiter.

APPENDIX III:

THE QUARTERNARIES OF THE TETRAKTYS

ON THE TETRAKTYS (Plato's model of The World Soul)

THE 11 QUARTERNARIES

1. 1st QUARTERNARY (Addition)	1+2+3+4=10
2. 2nd QUARTERNARY (Multiplication)	(1x1=1) (Unity) (2x3=6) (Side) (4x9=36) (Square) (8x27=216) (Cube) 1+2+3+4+8+9=27
3. 3rd QUARTERNARY (Magnitudes)	Point, Line, Surface, Solid
4. 4th QUARTERNARY (Elements)	Fire (1), Air (2), Water (3), Earth (4) 1 is to 2 as Fire is to Air. 1 is to 3 as Fire is to Water. 1 is to 4 as Fire is to Earth. 2 is to 3 as Air is to Water. 2 is to 4 as Air is to Earth 3 is to 4 as Water is to Earth.
5. 5th QUARTERNARY (Solids)	Fire=Tetrahedron/Pyramid Air=Octahedron Water=Icosahedron Earth=Cube
6. 6th QUARTERNARY (Created Things)	1=Seed (Point) 2=Length (Line) 3=Width (Surface) 4=Thickness (Solid)
7. 7th QUARTERNARY (Society)	Unity=Man Duad=Family Triad=Village Tetrad=City
8. 8th QUARTERNARY (Faculties)	Unity=Thought Duad=Science Triad=Opinion Tetrad=Feeling
9. 9th QUARTERNARY (Body/Soul)	Unity=Rationality Duad=Emotionality Triad=Will Tetrad=Body
10. 10th QUARTERNARY (Seasons)	Spring, Summer, Fall, Winter
11. 11th QUARTERNARY (Ages)	Childhood, Adolescence, Maturity, Old Age

"And the perfect world which results from these quarternaries is geometrically, harmonically and arithmetically arranged, containing in power the entire nature of number, every magnitude and every body, whether simple or composite. It is perfect because everything is part of it, and it is itself a part of nothing else."

Source: Theon of Smyrna (70?-135? CE)
Guthrie/Fiedler, *The Pythagorean Sourcebook and Library*, reproduced from *Mathematics Useful for Understanding Plato* by Theon of Smyrna, translated by Robert and Deborah Lawlor

APPENDIX IV:

THE COMPLETE TIMELINE OF THE LAMBDOMA

570 BCE	Approximate birth year of **Pythagoras**.
429 BCE	Birth year of **Plato**, who alludes to the Lambdoma in *Republic* (ca. 380 BCE) and *Timaeus* (ca. 360 BCE)
60 CE	Approximate birth year of **Nicomachus**, who writes the *Manual of Harmonics* and *Introduction to Arithmetic*, which deals primarily with the symbology of numbers
245 CE	Approximate birth year of **Iamblichus**, who writes a commentary on **Nicomachus**' *Arithmetic* which alludes to the Lambdoma
	(Here we have a 1600 year gap, during which time the Lambdoma was virtually unknown by anyone save perhaps the inner circle of Pythagoreans and Platonists, but to what extent it cannot be known. Works of **Iamblichus** and **Nicomachus** were only available in Greek, Latin or Arabic up until the eighteenth century.)
1806	Birth year of **Albert Freiherr von Thimus**.
1821	Birth year of **Hermann von Helmholtz**.
1863	Publication of **von Helmholtz's** *On the Sensations of Tone*
1868	Publication of **von Thimus'** *Die harmonikale Symbolik des Alterthums*
1878	**Albert von Thimus** dies
1891	Birth year of **Hans Kayser**
1894	**Hermann von Helmholtz** dies
1895	Birth year of **Ernst Levy**
1901	Birth year of **Harry Partch**
1914	Birth year of **Siegmund Levarie**
1918	Birth year of **Ernest McClain**
1920	Birth year of **Rudolf Haase**
1923	**Partch** discovers *On the Sensations of Tone* in the public library, and his ideas for just intonation are born
1925	Birth year of **Barbara Hero**
1928	**Partch** begins work on *Exposition on Monophony*
1934-35	**Partch** does research for *Exposition* at the British Museum

1945	Birth year of **Joscelyn Godwin**
1946	**Partch** builds the Diamond Marimba
1947	**Partch** completes work on his book, now with the title *Genesis of a Music*
1949	*Genesis of a Music* published
1950	**Hans Kayser** publishes *Lehrbuch der Harmonik*
1952	Birth year of **Werner Schulze**
1964	**Hans Kayser** dies
1967	**Rudolf Haase** founds the **Institute for Harmonic Research** in Vienna as part of the **Vienna College of Music and the Performing Arts**
1968	**Levy** & **Levarie** publish *Tone: a Study in Musical Acoustics*
1968	**Barbara Hero** discovers *Tone* at Boston Public Library, corresponds with **Levarie**
1974	*Genesis of a Music*, now out-of-print, republished in an expanded 2nd edition; **Partch** dies the same year
1981	**Ernst Levy** dies
1985	**Godwin** publishes *Cosmic Music*, which includes translations of articles by **Rudolf Haase**, including "Harmonics and Sacred Tradition," a source article for the Lambdoma
1987	**Godwin** publishes *Music, Mysticism, and Magic*, a sourcebook of writings on music from ancient to modern times, including works by **Plato**, **Plutarch**, **Plotinus**, **Iamblichus**, **Boethius**, **Zarlino**, **Kepler**, and **d'Olivet**
1992	**Schulze** becomes a professor at the **University of Music and Dramatic Arts-Vienna**, and the new head of the **Institute for Harmonic Research**
1993	**Godwin** publishes *The Harmony of the Spheres*, containing the only known English translation of any part of the **von Thimus**
1994	**Barbara Hero** presents the first **Pythagorean Lambdoma Harmonic Keyboard**
1997	**Hero** founds the **International Lambdoma Research Institute**
2002	The IHR becomes the **International Center for Harmonics** (IHZ), an independent entity from the Vienna University of Music and Performing Arts
2006	Publication of **Hans Kayser's** *Textbook of Harmonics* by Sacred Science Institute, first English language edition (translated by **Ariel Godwin**, son of **Joscelyn**)
2014	**Werner Schulze** due to retire from the University; future of the IHZ uncertain

APPENDIX V: THE 43-TONE SCALE OF HARRY PARTCH

Index No.	Ratio	Cents	Interval Name	12-TET Pitch
1	1/1	0	perfect unison	G
2	81/80	22	acute unison	↑G
3	33/32	53	augmented unison	G𝄲
4	21/20	84	small just minor second	↓G#
5	16/15	112	large just minor second	A♭
6	12/11	151	undecimal minor second	A𝄲
7	11/10	165	undecimal major second	-↓A
8	10/9	182	small just major second	↓A
9	9/8	204	large just major second	A
10	8/7	231	septimal major second	↑A
11	7/6	267	septimal minor third	↓B♭
12	32/27	294	Pythagorean minor third	B♭
13	6/5	316	just minor third	↑B♭
14	11/9	347	neutral third	B𝄲
15	5/4	386	just major third	↓B
16	14/11	418	undecimal major third	↑B
17	9/7	435	septimal major third	+↑B
18	21/16	471	subfourth	↓C
19	4/3	498	just perfect fourth	C
20	27/20	520	acute fourth	↑C
21	11/8	551	augmented fourth	C𝄰
22	7/5	583	small septimal tritone	↓C#
23	10/7	617	large septimal triton	↑D♭
24	16/11	649	diminished fifth	D𝄲
25	40/27	680	sub fifth	↓D
26	3/2	702	just perfect fifth	D
27	32/21	729	acute fifth	↑D
28	14/9	765	septimal minor sixth	-↓E♭
29	11/7	782	undecimal minor sixth	↓E♭
30	8/5	814	just minor sixth	↑E♭
31	18/11	853	neutral sixth	E𝄲
32	5/3	884	just major sixth	↓E
33	27/16	906	Pythagorean major sixth	E
34	12/7	933	septimal major sixth	↑E
35	7/4	969	septimal minor seventh	↓F
36	16/9	996	small just minor seventh	F
37	9/5	1018	large just minor seventh	↑F
38	20/11	1035	undecimal minor seventh	+↑F
39	11/6	1049	undecimal major seventh	F𝄰
40	15/8	1088	small just major seventh	F#
41	40/21	1116	large just major seventh	↑G♭
42	64/33	1147	diminished unison	G𝄲
43	160/81	1178	sub octave	↓G
44	2/1	1200	perfect octave	G

Referenced from Partch's *Genesis of a Music*, Dean Drummond, and the website of Bradford Blackburn carrying an excerpt from his dissertation "Partch's 43-Tone Scale" (http://www.bradfordblackburn.com/Dissertation/Partch's%2043-Tone%20Scale.pdf)

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